

# Catastrophe Bonds, Reinsurance, and the Optimal Collateralization of Risk Transfer\*

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## Abstract

Catastrophe bonds feature full collateralization of the underlying risk transfer, and thus abandon the insurance principle of economizing on collateral through diversification. Our analysis demonstrates that this feature of the catastrophe bond structure likely places limits on its ultimate penetration. However, we also show that fully collateralized instruments have important uses in a risk transfer market when insurers cannot contract completely over the division of assets in the event of insolvency, and, more generally, cannot write contracts with a full menu of state-contingent payments. In this environment, catastrophe bonds can sidestep reinsurance contracting constraints to deliver coverage to those most exposed to default. As such, catastrophe bonds can significantly improve welfare for segments of the insurance market, even though they may be unlikely to replace traditional reinsurance wholesale, absent major shifts in the financial landscape.

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# 1 Introduction

Recent disaster experience has produced a flurry of economic inquiry into catastrophe insurance markets. Especially puzzling is the apparent incompleteness of catastrophe risk transfer: The price of risk transfer seems high, risk is not spread evenly among insurers in the manner suggested by Borch's [2] ground-breaking theoretical result, and, in contrast to Arrow's well-known characterization of optimal insurance contracts, reinsurance consumers do not purchase coverage for high layers of risk. Froot [12] documents these puzzles and fingers various market imperfections as possible explanations.

Many have looked to securitization as a potential solution to these ills. Indeed, it is possible that the catastrophe-linked security (such as the catastrophe bond) will follow in the footsteps of the mortgage-backed security, eventually dominating its market and connecting those desiring protection with deep, unexploited pools of risk-bearing capacity in the capital markets. The logic is straightforward. If catastrophe risks are small relative to the capital markets and are, as the available evidence suggests, largely unrelated to the factors that drive performance on other securities, then the cost of bearing those risks should be small. Thus, once securitization distributes catastrophe risks to the broader capital markets, one might expect cost of catastrophe risk transfer to fall, and the dysfunction in today's market to dissipate.

The irony here is that capital markets have long been involved in supplying capital to bear catastrophe risks. The instruments of choice were and continue to be debt and equity capital on the balance sheets of insurance and reinsurance companies exposed to catastrophe risk. The relevant question is thus not whether things will change once the capital markets are

involved—since these markets are already “involved”—but the extent to which risk-bearing assets will be allocated between reinsurers issuing reinsurance policies and special purpose vehicles issuing catastrophe bonds. In other words, what role can the catastrophe bond be expected to play in the risk transfer market?

This paper examines this issue by developing a theory of risk collateralization. Specifically, we study the efficient division of risk-bearing assets between reinsurance company assets and catastrophe bond principal (both of which can be used to “collateralize” promises to indemnify consumers). Viewed in this light, the catastrophe bond at first blush looks less like a world-changing innovation and more like an atavism. Its current form features full collateralization and links principal forfeiture only to specific risks, thereby retreating from the time-tested concept of diversification that allows insurers to protect insured value far in excess of the actual assets held as collateral. In a world where frictional costs (e.g., due to taxes, regulations, or agency costs) make capital costly to hold, diversification lowers the cost of insurance. In this context, a fully collateralized instrument such as the catastrophe bond seems an unlikely competitor to traditional reinsurance products.<sup>1</sup>

In a narrow sense, economic theory supports the skeptical intuition outlined above. When reinsurance companies can write any type of contract with their insureds and frictional costs are identical for catastrophe bonds and reinsurer assets, catastrophe bonds are at best redundant, and at worst welfare-reducing. If the insurer is free to vary indemnity payments to consumers in every state of the world, it can engineer any possible menu of payouts through its own contracts.

However, contracting constraints and frictional cost differences create a significant and

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<sup>1</sup>Niehaus [23] observes this paradox on Page 593.

important role for catastrophe bonds. Typical reinsurance and insurance contracts do not specify *ex ante* rules for payouts in the event of insurer bankruptcy. Instead, the division of assets under bankruptcy is determined by insurance receivership laws, which distribute the assets of failed companies according to inflexible legal rules. Simple legal rules can, of course, be efficient under some circumstances: For example, when insureds are homogeneous and risk exposures are binary (loss or no loss), optimal insurance contracts are uniform, and simple rules—e.g., a *pro rata* rule that pays all claimants at the same rate on the dollar in the event of insurer bankruptcy—perform well. Under the more general case of consumer heterogeneity, however, *pro rata* rules misallocate assets in the bankruptcy state<sup>2</sup> by failing to direct sufficient indemnification to those who value consumption the most. Since some reinsurance buyers may be more exposed to default than others, those with greater exposure to bankruptcy risk may desire more security than their peers. Catastrophe bonds can help smooth out this misallocation, because their payouts are not subject to the solvency of an underlying insurer. By dedicating collateral to a specific insured, catastrophe bonds (and similar instruments) can improve welfare when reinsurers face constraints on the distribution of assets in bankruptcy *and* reinsure a heterogeneous group of risks. Catastrophe bonds can thus improve welfare in the presence of real-world contracting constraints.

Frictional cost advantages—such as those that might arise from differences in agency costs or taxation—could add to these “bankruptcy” advantages to expand the role of catastrophe bonds. In spite of any advantages, however, the catastrophe bond must ultimately reckon with its failure to economize on collateral. In particular, frictional cost and bankruptcy

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<sup>2</sup>Mahul and Wright [20] also note the inefficiency of *pro rata* rules in the context of a model with identical consumers but generalized loss distributions.

advantages must be weighed against the disadvantage of sequestering assets in a vehicle dedicated to a single consumer of risk protection—as opposed to making assets available to multiple buyers of protection, as in a reinsurance company. This forfeiture of diversification opportunities ends up acting as a headwind against catastrophe securitization as currently structured: The greater the diversification opportunities, the greater the frictional cost and bankruptcy advantages required for catastrophe bonds to take the lead in the risk transfer market.

Numerical simulation reveals that modest frictional cost advantages could vault catastrophe securitization into prominence within insurance market segments where diversification opportunities are few, or in those segments where buyers are unevenly exposed to bankruptcy risk. Conversely, however, market segments with substantial diversification opportunities would require extremely large frictional cost advantages, far beyond those seen today, for catastrophe securitization to make any headway. The analysis suggests that catastrophe bond issuance will improve performance in particular niches of the risk transfer market, but that widespread securitization of insurance exposures is unlikely absent dramatic reductions in the frictional costs associated with catastrophe bonds.

The paper is laid out as follows. Section 2 provides some background and context on catastrophe bonds. Section 3 then develops a concrete two-consumer example to illustrate the intuition behind our results. Section 4 develops our results formally in the context of a social planning problem with  $N$  consumers and offers numerical results to illustrate the trade-offs involved. Section 6 discusses other strategies for protecting consumers against default and interprets them in the context of the model. In particular, it considers how collateralization clauses in reinsurance policies influence the priority of claimants under bank-

ruptcy, and the extent to which such clauses substitute for fully collateralized instruments such as catastrophe bonds. Section 7 concludes.

## 2 Background and Motivation

A basic catastrophe bond transaction centers on a special purpose vehicle (SPV). The SPV sells securities (catastrophe bonds) to investors, and the proceeds from the sale are deposited in trust and invested. The SPV then provides reinsurance to a ceding insurer or reinsurer, who pays a premium in exchange. The premium, as well as income earned on the trust investments, funds interest payments to investors. If a contractually defined trigger event occurs, part or all of the bond principal is forfeited to the ceding company; if no event occurs, the principal is returned to investors.<sup>3</sup>

While early catastrophe bonds linked forfeiture of principal to the issuer's actual losses (an *indemnity trigger*), triggers linking forfeiture of principal to industry losses, catastrophe model output, or to specific parameters of the disaster (e.g., the strength of an earthquake centered in a certain geographic region) have grown in popularity. Some deals feature multiple event triggers—requiring two or more major disasters within a short time period to trigger principal forfeiture (see Woo [27]).

Catastrophe bond issuance in 2006 amounted to a record \$4.7 billion, with outstanding principal of \$8.5 billion.<sup>4</sup> Reinsurance side-car formation also surged, making 2006 a

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<sup>3</sup>For more details on the structure of insurance-linked securities, see, for example, Canabarro, E., Finke-meier, M., Anderson, R.R., and Bendimerad, F. (1998), "Analyzing Insurance-Linked Securities," Goldman Sachs & Co. Quantitative Research, October, 1998.

<sup>4</sup>Source: *The Catastrophe Bond Market at Year-End 2006: Ripples into Waves*, MMC Securities. Figures include only publicly disclosed transactions.

banner year for alternative risk transfer in the catastrophe market. On the other hand, catastrophe bond principal is tiny in comparison with the assets of the reinsurance industry. Despite being around for more than a decade, outstanding catastrophe bond principal in 2006 amounted to a tiny fraction of global insurance resources.<sup>5</sup>

Indeed, relative to the expectations of the 1990's, the volume of catastrophe securitization has been disappointing. The optimism in the past was buoyed by an tempting analogy with mortgage securitization:<sup>6</sup>

[Catastrophe bonds] herald a revolution in the insurance business. Even ordinary lines such as motor or health insurance will soon be turned into securities that investors can buy and sell in the capital markets. . . . Their effect will be increasingly to blur the distinction between insurance and investment banking. Instead of bearing risks themselves, insurers will concentrate on selecting risks and packaging them for sale to investors . . . Insurers are discovering what bankers know as securitisation: the process of assembling mortgages, credit-card receivables or even business loans into securities . . . Insurers are only now waking up to the potential benefits.

The “revolution” has not arrived yet, but the question of whether it is coming is more than just an academic curiosity. A revolution could dramatically change the policy calculus regarding government intervention in the insurance markets for terrorism and natural disasters. If the process of securitization is inevitable, and if that process dramatically alters the cost equation for primary consumers in affected areas, then it is conceivable that the private market will, in time, deliver significant increases in risk-bearing capacity and, ultimately, in the welfare of affected households. On the other hand, if securitization does not take off, or if the eventual result of securitization changes the form of capital deployment rather than the overall amount deployed, then purely private market solutions seem less promising.

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<sup>5</sup>Standard and Poor's *Global Reinsurance Highlights, 2006 edition* tallied over \$300 billion in total adjusted shareholder funds for the industry at year-end 2005. Including the resources of primary companies would skew the figures even further.

<sup>6</sup>“An Earthquake in Insurance,” *The Economist*, 2/28/1998.

Much hinges on what can be expected of catastrophe securitization now and in the future.

Our analysis of catastrophe securitization focuses on the issue of collateralization, which is a key difference between risk transfer using reinsurance and risk transfer using catastrophe bonds. Reinsurers accept risks from multiple consumers, and these risks typically aggregate to a figure well in excess of the assets held by the reinsurer to pay claims when they arise: The reinsurer relies on diversification within its portfolio of risks to deliver a high probability of meeting its obligations. The catastrophe bond structure, on the other hand, typically features a single consumer of protection, and it collateralizes the risk transfer in full: The assets in the special purpose vehicle are dedicated solely to protecting the consumer that sponsored the deal. The key outcome of interest, for our purposes, is how risk-bearing assets are to be allocated between the partially collateralized structures of traditional reinsurance institutions and the fully collateralized structures associated with securitization.

Forecasts of a dominant future for catastrophe securitization inevitably rest on the assumption that transfer through securitization will be more efficient than transfer through reinsurance, presumably because the frictional costs associated with catastrophe bonds will eventually be lower than those associated with reinsurance equity. While catastrophe bonds do seem to have frictional cost advantages, our analysis underscores the incompleteness of this argument: Assets in reinsurers support larger volumes of risk transfer by exploiting diversification opportunities within the risk transfer market. Because these opportunities are abandoned by fully collateralized structures, emerging frictional cost advantages in the latter will not necessarily translate into dominance in all segments of the market. Indeed, we offer numerical results later suggesting that, when significant diversification opportunities exist, frictional cost advantages have to be quite large for catastrophe securitization to

effectively compete with reinsurance as the primary vehicle for risk transfer. This is not to say that it is impossible for such advantages to emerge, nor to suggest that diversification opportunities are ever abundant: The point is simply that frictional cost advantages will be offset by diversification disadvantages, and it thus seems likely that catastrophe securitization will encounter stronger resistance to penetration than was seen in the case of mortgage securitization.

However, our analysis also shows that catastrophe bonds can serve an important economic role in *complementing* the reinsurance market—as opposed to replacing it. Full collateralization allows catastrophe bonds to be useful in cases where traditional risk transfer (e.g., through reinsurance policies) is subject to significant risk of counterparty default. In particular, we show how practical limits on reinsurance contract complexity interact with diversity of consumer risks and preferences to open up opportunities for catastrophe securitization.

### **3 A Simple Example**

In the context of a simple two-consumer example, we illustrate how, in the absence of frictional cost advantages, the role for catastrophe bonds depends on the presence of: (1) nonzero bankruptcy risk for the insurer; (2) contracting constraints that prevent the insurer from optimally allocating claims payments in the bankruptcy state; and (3) heterogeneity across consumers, such that one consumer faces greater exposure to insurer bankruptcy risk.

Consider the case of two consumers, named A and B. Consumer A faces a 10% chance of losing \$100, while Consumer B faces a 1% chance of losing \$100. An insurer issues simple contracts to indemnify the consumer, fully or partially, in the event of a loss. In

the bankruptcy state (where claims exceed insurer assets), claims payments are allocated according to a mechanical rule by dividing assets on a pro-rata basis, according to the claims made by the insureds.<sup>7</sup>

Suppose we have \$150 in assets. How should we allocate them? Consider first the case where we use all \$150 to fund an insurance company, which issues a \$100 limit insurance policy to A and a \$100 limit policy to B. Expected claims in this example equal:

$$10\% * \$100 + 1\% * \$100 = \$11 \tag{1}$$

The insurer is able to pay all claims in full except when both consumers suffer a loss; in that event, the insurer pays out all \$150 of its assets but declares bankruptcy. Therefore, expected claims payments equal:

$$10\% * 99\%(\$100) + 1\% * 90\%(\$100) + 10\% * 1\%(\$150) = \$10.95 \tag{2}$$

Overall, the insurer pays  $\$ \frac{10.95}{11}$ , or better than 99 cents, on the dollar. However, the two consumers are unevenly exposed to default. Consumer B ends up being much more exposed to bankruptcy risk on a per dollar basis, because she faces a higher relative likelihood of suffering a loss in the state of the world where the other consumer *also* suffers a loss.

Specifically, Consumer A expects to lodge \$10 worth of claims and to receive payments of:

$$10\% * 99\% * \$100 + 10\% * 1\% * \$75 = \$9.975 \tag{3}$$

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<sup>7</sup>The exact form of the mechanical rule is less relevant than the presence of contracting constraints in the bankruptcy state.

On the other hand, Consumer B expects to lodge \$1.00 worth of claims, but receive

$$1\% * 90\% * \$100 + 1\% * 10\% * \$75 = \$0.975 \quad (4)$$

Thus, Consumer A receives 99.975 cents on the dollar, while Consumer B receives only 97.5.

Consumer A is better insured than Consumer B, and we might consider redistributing coverage from Consumer A to Consumer B. One way of accomplishing this is to redeploy some of our assets in the form of a catastrophe bond tied to Consumer B. Suppose we now use \$100 to fund the insurance company, which sells a \$100 limit insurance policy to Consumer A and a \$50 limit policy to Consumer B. We then use the remaining \$50 on a catastrophe bond payable to Consumer B in the event of a loss.

Consumer A still expects to lodge \$10 worth of claims, but now receives payments of:

$$10\% * 99\% * \$100 + 10\% * 1\% * \left(\frac{100}{150} * 100\right) = \$9.967 \quad (5)$$

On the other hand, Consumer B now expects to lodge \$0.50 worth of claims with the insurance company, but now also is entitled to receive \$50 of catastrophe bond principal in the event of a loss:

$$1\% * 90\% * (\$50 + \$50) + 1\% * 10\% * \left(\frac{50}{150} * \$50 + \$50\right) = \$0.983 \quad (6)$$

The recovery differential has narrowed. Consumer A now receives 99.67 cents of relief per dollar of loss, a slightly worse rate than before. With the catastrophe bond in place, Consumer B now receives a bit more—98.3 cents.

In this example, using the catastrophe bond instead of a full reinsurance solution effectively transfers coverage from one consumer to the other. The transfer occurs only when the reinsurer defaults: We have sufficient assets to fully indemnify both consumers *except* when both experience a loss, and the catastrophe bond allows us to affect the distribution of indemnification in that unfortunate state of the world. Of course, the question of whether or not this redistribution is desirable depends on particulars such as preferences—but the general point is that the allocation of assets to consumers in the bankruptcy state may be suboptimal in a pure reinsurance solution, and the catastrophe bond is one way of securing the interests of one consumer over the other.

The presence of contracting constraints, the risk of bankruptcy, and the presence of consumer heterogeneity all play key roles in driving this result. If an insurer is able to write complex contracts that vary indemnification across all states of the world, it can replicate the payout structure of a catastrophe bond without using the bond itself. For instance, in the example above, we could replicate the payoffs involved under the second approach (using the catastrophe bond) simply by capitalizing the insurer with \$150 and issuing policies offering full \$100 indemnification except in the case where both consumers had losses, in which case Consumer A would receive \$66.67 and Consumer B would receive \$83.33.<sup>8</sup> Contracting constraints that prevent the insurer from specifying complicated priority rules under bankruptcy are necessary to preclude this possibility. Heterogeneity also plays an

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<sup>8</sup>Note that if we allowed policy limits to exceed insurer assets, this would allow insurers to influence the division of resources in the bankruptcy state. However, this is a blunt instrument for resource allocation that cannot generally replicate the payouts of catastrophe bonds. For example, with more than two consumers, insurer bankruptcy is not perfectly correlated with the loss experience of any one consumer, because there are many possible loss configurations that trigger bankruptcy. Nevertheless, in the general theory developed in Section 4, we place no constraints on the choice of policy limits.

important role by rendering mechanical bankruptcy rules inefficient. If Consumers A and B were identical, an equal pro rata division of resources in the bankruptcy state would be optimal, and neither consumer would be any more exposed to default risk.

This example shows how catastrophe bonds can be used to improve social welfare by redistributing coverage among consumers in “unfortunate” states of the world, but it falls short of illustrating other aspects of the general trade-off between catastrophe bonds and reinsurance. Earlier, we emphasized the costliness of fully collateralized catastrophe bonds, relative to less than fully collateralized insurance. Yet in this example, there is no disadvantage to “sequestering” capital in the form of a catastrophe bond since we make full use of the collateral assets. In the general characterization of the problem explored below, an important drawback associated with the catastrophe bond is that the assets are dedicated to one consumer and not available to pay losses experienced by others.

## 4 Theory

Our approach borrows from Borch’s analysis of optimal risk sharing among many consumers: Instead of modeling individual behavior, we study the social planning problem. This approach allows us to sidestep the thorny issues involved with insurance company pricing in the presence of heterogeneity (see Phillips et al. [24], Myers and Read [22], and Zanjani [28])

Consider a world with  $N$  consumers. Consumer  $i$  is endowed with initial wealth  $W_i$  and faces the risk of experiencing a loss of fixed size—denoted by  $L_i$ . To characterize the possible states of the world, we define a row vector  $\mathbf{x}$  of length  $N$ , with the elements all taking a value of zero or one:  $\mathbf{x}(i) = 1$  means that consumer  $i$  experienced a loss, while  $\mathbf{x}(i) = 0$  means

that she did not. Let  $\Omega$  denote the set of all such vectors of length  $N$  with the elements taking values of one or zero. Each element of  $\Omega$  corresponds to a complete description of one possible state of the world. The entire set  $\Omega$  contains all possible such states. The following set definitions are useful:

$$\Omega^i = \{\mathbf{x} : \mathbf{x}(i) = 1\},$$

the set of all states in which agent  $i$  suffers a loss, and

$$\Gamma(\mathbf{x}) = \{i : x(i) = 1\},$$

the set of all agents that suffer a loss in state  $\mathbf{x}$ .

Thus, using this notation, we may describe the probability of loss faced by consumer  $i$  as:

$$p_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x})$$

The consumers are risk averse,<sup>9</sup> and there are two risk transfer technologies available. First, we can set up an insurance company and issue insurance policies to consumers, collateralized by the assets of the company. Second, we can issue a risk-linked security on behalf of a consumer (i.e., a catastrophe bond) that pays off in the event that the consumer experiences a loss.

The insurance company is formed with assets of  $A$ . Throughout our discussion, we think

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<sup>9</sup>In the case where the consumers are insurance companies or other financial institutions, risk aversion can be motivated by Froot and Stein [13]

of “assets” as all the resources the insurer can use to pay claims. Therefore, it includes both capital paid in by investors and premiums paid in by consumers. For our purposes, the key issue is whether or not a given dollar is available for claims payment—not how it would be treated by accounting conventions. In the event of a loss (or losses), the consumers can draw on the assets to pay claims. When assets exceed claims, what remains after claims are paid reverts to investors. On the other hand, if claims exceed assets, the company defaults, and claimants are assumed to be paid according to a pro rata rule—everyone receives the same rate of recovery per dollar of claim.

Each consumer can separately issue a catastrophe bond<sup>10</sup> to investors. The principal of the bond is forfeited to the consumer in the event of a loss, but not otherwise. Let  $B_i$  be the bond issuance of consumer  $i$ . We simplify matters by assuming *indemnity* triggers—where principal forfeiture is linked directly to issuer loss experience. Hence, we avoid the complexities of optimal trigger design (see Doherty and Mahul [8]) and the problem of basis risk. We do not directly model these and other costs associated with asymmetric information,<sup>11</sup> but such costs can be thought of as being embedded in the frictional costs associated with catastrophe bond principal described below.

What do insurance policies and catastrophe bonds cost? The cost of risk transfer can be decomposed into 1) fair ex ante compensation for claims expected to be paid under the risk transfer agreement and 2) frictional costs associated with establishing and maintaining the risk transfer scheme. We start simply by assuming that the cost of risk transfer amounts to

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<sup>10</sup>We refer to “catastrophe bonds” because of their familiarity, but the analysis that follows obviously applies to other highly collateralized instruments used in risk transfer—such as collateralized reinsurance policies and “side-cars.” These and other risk transfer alternatives will be discussed in Section 6.

<sup>11</sup>See Brandts and Laux [4] for a theoretical justification for the catastrophe bond market based on asymmetric information between insurers and reinsurers.

the expected value of claims plus a frictional cost proportional to the amount of collateral used in the risk transfer scheme (i.e., the amount of assets used in the insurance company, or the amount of catastrophe bond principal used). We then generalize the results to the case where fair compensation for claims risk reflects the correlation between claims and other capital market assets.

We start with the case where frictional costs are the only true costs of insurance provision, because such costs turn out to be the critical determinants of capital allocation. In the absence of frictional costs, all consumers will be fully insured, and it is irrelevant how risk transfer technologies are combined in providing this full insurance: Frictional costs provide a motivation to economize on capital in the process of collateralizing risk-transfer. Accordingly, we start with a model where insurance risks are “zero beta,” but where frictional capital costs lead to limited risk transfer. We then show that these results hold even when insurance risks correlate in some way with capital market returns (and are priced accordingly).

Frictional costs are imagined here as deriving from agency costs, taxes, liquidity costs, or other frictions. Each dollar of assets held in the insurance company results in per unit frictional costs of  $\delta_A$ . Each dollar of catastrophe bond principal raised has the frictional cost  $\delta_B$ . There are many reasons why we might expect the frictional costs associated with the two risk transfer technologies to differ. Insurance company assets are under the discretion of company managers, who may deploy assets suboptimally from the perspective of investors and/or consumers. Catastrophe bond principal, on the other hand, is largely insulated from the discretion of management (especially where non-indemnity triggers are involved): Moreover, the distribution of interest and the return of capital takes place at contractually

specified dates, making the investment of finite duration with predictable returns. However, the protection of investors is purchased at the expense of consumers, who may be exposed to basis risk. In any case, while the micro-foundations of frictional costs are a potentially interesting topic, it is not our goal to explicitly model them here: Instead, we take as given the proposition that the two technologies have different frictional costs, and we explore the optimal structure of the risk transfer market on the basis of that assumption.

Consumers must pay for all frictional costs and fair compensation for recoveries expected from the risk transfer. Denote the portion of this total risk transfer cost allocated to consumer  $i$  as  $c_i$ .

Insurance policies are represented as simple promises of indemnification: The insurer promises to pay  $I_i$  in the event that consumer  $i$  experiences a loss. We place no constraints on the promised indemnity: It may be less than, equal to, or greater than the prospective loss. However, contracting constraints come into play in the sense that we do not allow the insurer to offer a schedule of promised indemnification, with the amount contingent on the loss experiences of other insureds. If the insurer is able to pay, it pays in full; if not, it defaults, and all claims are paid at the same rate on the dollar. The example of Section 3 suggests that relaxing this contracting constraint will obviate roles for catastrophe bonds or other fully collateralized instruments. In the Appendix (Section A), we verify that this is in fact the case: When frictional costs are identical ( $\delta_A \equiv \delta_B$ ), allowing the social planner to arbitrarily vary the indemnification promised in each insurance policy eliminates any potential role for catastrophe bonds.

We can now define utility for consumer  $i$  (according to the usual Von Neumann-Morgenstern assumptions) as:

$$EU_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i(W_i - L_i + f_{\mathbf{x}} I_i + B_i - c_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i(W_i - c_i), \quad (7)$$

where  $f_{\mathbf{x}}$  represents the proportion of the indemnity payment actually paid in state  $\mathbf{x}$ .

The social planning problem can now be written as:

$$\max_{A, \{B_i\}, \{c_i\}, \{I_i\}, \{f_{\mathbf{x}}\}} V = \sum_i EU_i \quad (8)$$

subject to:

$$[\mu] : \sum c_i \geq \delta_A A + \delta_B \sum_i B_i + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} f_{\mathbf{x}} I_i + \sum_{i \in \Gamma(\mathbf{x})} B_i \right) \quad (9)$$

$$[\lambda_{\mathbf{x}}] : f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \leq A, \forall \mathbf{x} \quad (10)$$

$$[\phi_{\mathbf{x}}] : f_{\mathbf{x}} \leq 1, \forall \mathbf{x} \quad (11)$$

$$[\Lambda_{\mathbf{x}}] : (1 - f_{\mathbf{x}}) \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) \leq 0, \forall \mathbf{x} \quad (12)$$

and subject to non-negativity constraints on catastrophe bond principal and policy limits.

Constraint 9 ensures that consumers' total payments for risk-transfer instruments ( $\sum c_i$ )

cover the frictional costs of capital and expected losses. The remaining constraints are

contracting constraints. Constraint 10 ensures that the insurer always has enough assets

on hand to cover *actual* (as opposed to promised) liabilities. Constraint 11 precludes the

insurer from ever paying out more than the policy limit. Finally, constraint 12 ensures that

the insurer always pays claims in full, whenever it remains solvent.

The optimality conditions are derived in the Appendix (Section B.1), as is the following

marginal condition for catastrophe bond issuance (where we use the notation  $U_i^{\mathbf{x}}$  to denote the utility of consumer  $i$  in state  $\mathbf{x}$ ):

$$R_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} \right) - \sum_{\mathbf{x} \notin \Omega^i} \sigma_{\mathbf{x}} + (\delta_A - \delta_B)\mu \leq 0 \quad (13)$$

where  $\frac{\partial U_i^{\mathbf{x}}}{\partial W}$  is the marginal utility of wealth for consumer  $i$  in state  $\mathbf{x}$ , and:

$$w_j^{\mathbf{x}} \equiv \frac{I_j}{\sum_{j \in \Gamma(\mathbf{x})} I_j},$$

$$\sigma_{\mathbf{x}} \equiv \lambda_{\mathbf{x}} - (1 - f_{\mathbf{x}})\Lambda_{\mathbf{x}}.$$

$\sigma_{\mathbf{x}}$  is the marginal value of insurance company assets in state  $\mathbf{x}$ . This value will be zero in states where the company remains solvent.

$R_i$  is the marginal value of catastrophe bonds.  $R_i < 0$  if and only if catastrophe bonds cannot improve on a reinsurance-only equilibrium. Specifically, if  $R_i$  is negative, this means that catastrophe bond issuance was not useful (optimal) for consumer  $i$ —or, in other words, that  $B_i^* = 0$ .

We start analysis of (13) by considering the case where

$$\delta_A \equiv \delta_B \equiv \delta.$$

Thus, we initially focus on how the nature of preferences and risk affect the optimal mix of the two risk transfer technologies. This reveals three noteworthy results.

First, a catastrophe bond's potential to enhance the welfare of the issuing consumer is

intimately linked to the presence of default risk. If consumer  $i$  does not face any risk of default (i.e.,  $f_{\mathbf{x}} = 1$  for all  $\mathbf{x} \in \Omega^i$ ), catastrophe bond issuance will not be useful for that consumer.<sup>12</sup>

Second, assuming consumer  $i$  is confronted with default, catastrophe bond issuance on behalf of that consumer may be useful only if her marginal valuation of consumption in the states where the company defaults on her claim exceeds the average valuation of other consumers who lose in those same states:

$$\sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} \right) > 0.$$

Intuitively, it will not make sense to dedicate collateral to consumer  $i$  if that collateral is actually worth more to her companions in states of default. If her companions value that collateral more highly on average than she does, it will be more efficient to add that collateral to the insurance company or to issue catastrophe bonds on behalf of the high valuation consumers.

Finally, the value of catastrophe bond issuance for consumer  $i$  also depends on the extent of diversification possibilities, as captured in:

$$\sum_{\mathbf{x} \notin \Omega^i} \sigma_{\mathbf{x}}.$$

If that term is positive, it means those diversification possibilities still exist. In other words,

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<sup>12</sup>This is equivalent to saying that  $R_i < 0$ , except in solutions where the insurance company never defaults on any contract. If the insurer never defaults,  $R_i = 0$ , implying that catastrophe bonds could figure in a solution. However, as shown in the Appendix (Section B.2), any such solution would not be unique: Without default, any solution with catastrophe bond principal can be matched by a solution without catastrophe bond principal.

the company is defaulting in states of the world where consumer  $i$  does not experience a loss, and, thus, there are other consumers who would enjoy benefits from increasing the capitalization of the insurance company. While this does not preclude the solution from featuring catastrophe bond issuance on behalf of consumer  $i$ , it makes it more difficult for the catastrophe bond to be the preferable instrument for addressing the the risk transfer needs of the consumer in question: Any benefits obtained by sequestering collateral on behalf of consumer  $i$  (and thus shielding the assets from consumers who place lower valuations on additional coverage in those states where consumer  $i$  is exposed to default) must be weighed against the cost of preventing consumers exposed to default in other states of the world from accessing that collateral.

If bond principal enjoys frictional cost advantages relative to insurer assets ( $\delta_B < \delta_A$ ), additional opportunities for catastrophe bond issuance arise. Note, however, that such frictional cost advantages (to the extent they exist) are by no means the only consideration in assessing the potential for the catastrophe bond market. The extent of heterogeneity across consumers—in terms of preferences and in terms of risk exposures—as well as the presence or absence of diversification opportunities, still figure in the calculus of catastrophe bond issuance.

The importance of differences across consumers is highlighted by the case where consumers are homogenous, or ex ante identical. The Appendix explores this in detail, showing that catastrophe bonds will not be useful in this case unless they possess frictional cost advantages. The result (shown in the Appendix - Section B.3) can be understood by noting that, assuming that symmetric solutions apply under homogeneity, the marginal utilities of consumers who lose will be equivalent in each state. So equation 13 reduces to:

$$R_i = - \sum_{x \notin \Omega^i} \sigma_{\mathbf{x}} + (\delta_A - \delta_B)\mu \leq 0$$

In other words, catastrophe bond issuance will be strictly suboptimal in the absence of frictional cost advantages unless diversification possibilities have been completely exhausted.

In mathematical terms, cat bonds are strictly welfare-reducing, unless  $\sigma_x = 0$  for all  $x \notin \Omega^i$ .<sup>13</sup>

This condition can hold only when all consumers enjoy full indemnification except in the state where everyone experiences a loss. Thus, under homogeneity, cat bonds can be used only if the risk of insurer default is confined to the absolute worst-case scenario of  $N$  losses.

The  $N$ -consumer case under homogeneity exposes the disadvantage of catastrophe bonds with respect to diversification. Even when catastrophe bonds are cheaper than insurance company assets (i.e., if  $\delta_B < \delta_A$ ), they could still be strictly suboptimal if the welfare-maximizing solution involves tolerance of default beyond the absolute worst case scenario of  $N$  losses.

To this point, we have abstracted from modeling any risk premium that might be demanded by investors. However, the compensation owed to investors will generally depend on how the insurance risks borne correlate with returns on other capital assets. We explore this possibility in the Appendix (Section C.1). Importantly, we show that frictional costs are key. In particular, if frictional costs are zero (and the cost of risk transfer is driven entirely by the equilibrium compensation for risk-bearing reflected in the securities markets), then the optimal collateralization structure is indeterminate: All consumers will fully insure, and

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<sup>13</sup>Consistent with the insurance demand model of Doherty and Schlesinger [9], the consumer has less than full coverage in the presence of default risk. By reducing the premium paid, partial coverage transfers wealth into states of the world where the company is insolvent.

it does not matter how insurance policies and catastrophe bonds are combined in providing that protection. However, if frictional costs are present, the earlier results still carry through—even after the introduction of security markets and equilibrium risk pricing.

## 5 Empirical Implications of the Theory

Our results flow from a fundamental trade-off between reinsurance’s inefficiencies due to bankruptcy and its economies of collateral. This trade-off has a variety of specific aspects. To illustrate, we obtain numerical solutions for the case of constant absolute risk-aversion (CARA), and show how these vary with opportunities for diversification, bankruptcy exposure, and frictional cost differences. The simulations characterize the financial conditions under which bond issuance improves welfare. We then present some statistics on frictional costs and collateralization in today’s market, to shed light on the prospects for widespread securitization of insurance contracts.

The numerical analysis demonstrates that — given today’s levels of collateralization and frictional cost advantages — securitization is unlikely to supplant traditional insurance contracts in widespread fashion. However, it also suggests that securitization can substantially improve welfare for particular market segments subject to extraordinarily high degrees of implicit collateralization in the reinsurance market. These include segments with risks that are particularly correlated and hard to diversify, along with segments where heterogeneous insureds are unevenly exposed to bankruptcy risk.

The costs and benefits of catastrophe bonds can be studied along two key axes: frictional costs, and the collateralization of insurance. When bonds have frictional cost advantages,

they are more likely to be issued, but the requisite size of that cost advantage depends on the economies of capital achieved by insurers. When frictional cost advantages are modest, catastrophe bonds have value for segments of the market where reinsurance transfer is heavily collateralized.

Figures 1 and 2 shed light on the prospects for widespread securitization of risk. Figure 1 considers the case of homogeneous consumers, and varies: (1) Frictional cost advantages for catastrophe bonds (x-axis); (2) Number of consumers (N); and (3) probability of loss (p). The frictional cost advantage on the x-axis represents the percentage cost advantage for catastrophe bonds. For example, a value of 40% implies that  $\frac{\delta_A - \delta_B}{\delta_A} = 0.4$ .

The figure shows that catastrophe bonds are not used in the absence of a frictional cost advantage over reinsurance company assets—as predicted by theory. As the frictional cost advantage increases, however, so does the propensity to use catastrophe bonds. Moreover, if the frictional cost advantage is large enough, catastrophe bond principal can “dominate” in the sense of comprising the majority of risk-bearing assets in the risk transfer market.

The frictional cost advantage required tends to rise with the number of insureds, but fall with the probability of loss. That is, catastrophe bonds are most advantageous when there are few consumers and a high probability of loss. As we will show shortly, fewer consumers and higher loss probabilities imply fewer opportunities for diversification. For example, the case of ten insureds and a 10% loss probability features catastrophe bond participation starting at a frictional cost advantage of about 10% and passing reinsurance when the advantage exceeds 25%. With 50 insureds and a 1% probability of loss, however, catastrophe bond participation doesn’t even start until the advantage exceeds 80%.

The role played by collateralization becomes apparent in Figure 2, which provides a scat-

ter plot relating the frictional cost advantage necessary for catastrophe bond participation (the  $y$ -axis) with the extent of collateralization present in an insurance-only solution (the  $x$ -axis) for various combinations of  $N$  (number of consumers),  $p$  (probability of loss), and  $r$  (coefficient of absolute risk aversion).

On the  $x$ -axis is the extent of collateralization present before catastrophe bonds are introduced (i.e., the fraction of total exposure  $N * L$  that is covered by assets in the insurance company). The  $y$ -axis gives the frictional cost advantage required for any catastrophe bond issuance to be optimal. The points in the plot correspond to different configurations for the number of insureds ( $N$ ), the coefficient of absolute risk-aversion ( $r$ ), and loss probabilities. The values for  $N$  and  $r$  are given in the legend; the loss probabilities range from 0.01 to 0.20.

The figure demonstrates that a high degree of collateralization within the reinsurance market reduces the cost advantage required for catastrophe bond penetration. The intuition can be grasped by imagining the polar case where reinsurance company assets completely cover total exposure, so that the risk transfer is completely collateralized. Since reinsurance has no collateralization advantage in this case, the catastrophe bond requires only an epsilon advantage in frictional costs. In circumstances close to that polar case—where the optimal collateralization level is high—diversification opportunities are limited, so catastrophe bonds do not require much of a frictional cost discount to be deployed. On the other hand, when total exposure exceeds collateral assets by a substantial margin (e.g., the upper left quadrant of Figure 2), diversification opportunities are substantial, and catastrophe bonds are useful only in the presence of substantial frictional cost advantages.

Thus, collateralization is lowest in cases where events are relatively rare and insureds relatively numerous. Figure 2 illustrates that the degree of collateralization is a useful

summary measure of these factors. Therefore, insurance market segments with higher degrees of collateralization can be expected to deploy catastrophe bonds sooner and in larger quantities than their less collateralized counterparts if and when frictional cost advantages materialize.

So are we headed toward a world where individual homeowners and auto exposures are securitized and traded in the same manner as credit card receivables and auto loans? Figures 1 and 2, when applied to real world data,<sup>14</sup> suggest that such an outcome is highly unlikely. On an aggregate basis, collateralization in the US property and casualty industry is less than 5%. To illustrate, AIR Worldwide Corporation has estimated total US property exposure at \$44 trillion,<sup>15</sup> while the total claims-paying resources of the U.S. insurance and global reinsurance industries is approximately \$2 trillion. Even if all these resources were dedicated to U.S. property, this yields a rate of collateralization under 5%. The actual figure would drop even further when liability and other exposures are added in. According to Figure 2, a homogenous market with this low level of collateralization would require an enormous frictional cost advantage—more than 90%—for catastrophe bonds to even be issued. Even our most generous estimates of the actual frictional cost advantage of catastrophe bonds do not approach this level. As detailed in Appendix D, we estimate the annual frictional cost associated with catastrophe bond principal to be 600 basis points, while rough estimates of the corresponding frictional costs associated with reinsurance assets and with total domestic insurance industry assets to be about 1200 and 2000 basis points, respectively. While such advantages (if they are in fact so large—see below) are significant, diversification

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<sup>14</sup>The details supporting the empirical estimates used in this section are provided in Appendix D

<sup>15</sup>AIR Worldwide Corporation (2006), *The Coastline at Risk: Estimated Insured Value of Coastal Properties*. Boston: AIR Worldwide.

is so value-enhancing within the overall insurance market that a widespread transition to a model where insurers originate and transfer primary exposures to fully collateralized special purpose vehicles—seems very far from being cost-effective.

The aggregate data also cast doubt on the viability of such a model in catastrophe-prone areas. The same AIR study estimates \$7.2 trillion of insured property in coastal areas exposed to hurricanes in the United States, a figure also far in excess of the total assets that could plausibly be reckoned to be supporting those exposures. Evidently, catastrophe exposure on most properties is largely retained by underwriters and managed through diversification, even if incompletely. Moving to a system of full collateralization would impose costs that exceed the likely reductions in frictional cost, with the resulting rise in the cost of risk-management likely being borne by the primary insureds.

However, while it may never make sense to securitize the vast majority of primary exposures, catastrophe securitization could still have a significant impact in the “secondary” risk transfer market. In particular, there are segments of the reinsurance market where diversification opportunities are limited, and these segments may feature high levels of implicit collateralization by reinsurers. It is possible that catastrophe bonds may improve welfare in such segments. Figure 3 illustrates this point. Figure 3 analyzes the role played by correlated risks, which increase collateral requirements in reinsurance by reducing opportunities for diversification; the relative payoff to securitization increases as a result. We analyzed scenarios with 50 homogenous consumers each facing a 10% chance of losing \$50, and with a small (10%) frictional cost advantage for catastrophe bonds. The correlation between consumer losses is allowed to vary from zero to one. The solid line in the figure shows the fraction of assets deployed in catastrophe bonds, rather than in an insurance company. As

the correlation rises, so does the fraction of capital devoted to catastrophe bonds, while the fraction devoted to traditional insurance falls. Catastrophe bonds comprise a majority of the capital market when the correlation approaches 0.2.

The dashed line in the figure illustrates why. This line shows insurer assets as a fraction of insurer loss exposure, in an insurance-only solution. It represents the degree to which the insurer is able to economize on collateral. As the correlation approaches 0.2, insurers are holding assets equal to 80% of exposure, and thus cannot be gaining much through diversification. In this example, insurance holds only a slight edge in collateralization over the catastrophe bond—even at modest levels of correlation. This advantage is overcome by the bond’s modest frictional cost advantage. Moreover, observe that bond issuance is zero until the optimal insurance solution approaches a 70% degree of collateralization, very high by any measure.

While it is difficult to observe the level of collateralization within the reinsurance market alone, an examination of property reinsurance pricing (see the Appendix) illustrates where catastrophe securitization is likely to benefit insureds. On the one hand, regional reinsurance programs seem well-diversified, and thus poor candidates for catastrophe bond penetration. To illustrate, recent data on the cost of regional reinsurance programs most closely comparable to catastrophe bond deals<sup>16</sup> ranged from 300 basis points per unit of exposure in 2005 (pre-Katrina) to 450 basis points after Katrina. A plausible explanation of these relatively narrow spreads is that the reinsurance industry is able to effectively diversify regional risks and thus assigns relatively little capital to support them: Since little capital is assigned to

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<sup>16</sup>The data are for regional programs (as defined by Guy Carpenter) with a 2% rate of expected loss per unit of exposure.

support the risk transfer, the frictional costs of equity do not contribute burdensome margins to the price. Furthermore, the cost to the buyer is well below the 600 basis point frictional cost associated with catastrophe bonds—and the 600 point figure is an average across a sample of deals, with an expected loss of 1.34%: When deals are done in layers of risk that are more likely to be tapped, the cost rises—above 1000 basis points for the subsample of deals with expected loss of 2% or more. Indeed, experience with corporate bonds more generally suggests that the risk transfer through catastrophe bonds will become much more expensive at higher levels of risk (Altman [1]).

Pricing for national property reinsurance programs, however, looks much different. In 2005 (pre-Katrina), pricing for programs with an expected loss of 2% was around 550 basis points, while the average figure for January 1 renewals in 2006 and 2007 was around 1300 basis points. These programs are evidently much more capital intensive in the sense that they are priced as if the reinsurers are assigning much more capital to them—even though they have the same expected loss as their regional counterparts. Evidently, the limits of diversification are being reached in the risk being transferred by national carriers—and it is in this area and those beyond it (e.g., property retrocessions) where one can start making a case that catastrophe securitization may prove to be competitive, especially during hard markets.

Figure 4 illustrates another important case where bond issuance may be significant even in the absence of heavy collateralization within reinsurers. If particular subgroups are unevenly exposed to bankruptcy, bonds provide a direct way of securing their interests. For these scenarios, frictional costs are identical for catastrophe bonds and insurer assets. Differential exposure to bankruptcy generates a motive for bond issuance. We divide the set of consumers

into two halves: one half faces a uniform loss probability of 1%; for the other half, we allow the loss probability to vary along the x-axis. When loss probabilities are identical, or close to identical, no bonds are issued. However, when the “high-risk” types face loss probabilities over 6%, compared to 1% for the low-risk types, some bond issuance takes place.

The solid curve marked by diamonds illustrates the fraction of risk-bearing assets deployed in bonds, while the solid curve marked by circles gives the fraction of insured value produced by bonds. Catastrophe bonds represent a majority of risk-bearing assets when the difference in loss probability approaches an order of magnitude. At this point, they also produce a majority of the insured value *for the low-risk types*. However, because they are so much more heavily collateralized than insurance (see the dashed line), they still do not produce most of the aggregate insured value. Catastrophe bonds are only issued to the low-risk types, and thus never account for more than half the insured value in the overall market.

This figure emphasizes our finding that catastrophe bonds can have exceptional value for niches of the market that are unevenly exposed to the risk of insurer default. Therefore, even if the industry as a whole is financially healthy, well-diversified, and not unduly collateralized, segments of the market may benefit from the security offered by catastrophe bond issuance.

## 6 Other Risk Transfer Options

To this point, we have limited our attention to catastrophe bonds and traditional reinsurance policies. With this focus, we risk overlooking hedging strategies based on other risk transfer options that could potentially yield welfare improvements. In this section, we consider how

other risk transfer strategies fit into our framework.

## 6.1 Collateralized Reinsurance and Side-cars

Reinsurance can be “collateralized” in at least two senses. The first, more common sense, is a contract clause requiring the reinsurer to collateralize claims obligations at the time they are incurred but before they are due to be paid. The second is a full or partial collateralization of the policy limits at the inception of the contract. Both are interesting for our paper because they allow the reinsurer and its customers to influence how assets are divided up in the event of bankruptcy. We discuss each in turn.

Reinsurance contract clauses regarding the collateralization of liabilities arise in transactions between offshore reinsurers and U.S. cedents (i.e., buyers of reinsurance). Regulations regarding statutory credit for reinsurance typically stipulate that an insurer may take credit for anticipated recoveries from unlicensed reinsurers only if those anticipated recoveries are fully secured. Acceptable forms of security include funds held in trust and clean, irrevocable, and evergreen letters of credit issued by financial institutions deemed acceptable by the regulator in question.

To the extent that some cedents have these contract clauses and others do not, the clauses may be interpreted as a means of affecting the distribution of assets in bankruptcy. Secured claimants effectively “step ahead” of unsecured claimants in the liquidation process. However, it should be noted that the ability to “step ahead” is by no means absolute and depends on ex post actions by the insurer. For example, a transfer of assets to a trust for the benefit of a cedent (or to collateralize a letter of credit issued by a third-party for the

benefit of a cedent) can be challenged as a voidable preference if bankruptcy follows soon thereafter.<sup>17</sup> Hence, in practice, a cedent cannot count on security being posted when a reinsurer is in or near insolvency, and, even if the reinsurer is willing to post security, the transfer is subject to challenge.

It is also possible to provide partial or full collateralization (e.g., a letter of credit) of the policy limits at contract inception. This approach is useful if the underwriter does not have a financial strength rating. Another variation on this theme is the reinsurance “side-car,” where investors capitalize a special purpose company to provide quota-share reinsurance to a ceding company, with the capital being held in a collateral trust account for the benefit of the cedent.<sup>18</sup> In our framework, these forms of collateralized reinsurance are similar, from the perspective of collateralization, to a catastrophe bond with an indemnity trigger.

For underwriters issuing policies to multiple cedents that differ in the degree of collateralization, the situation is more complicated. Since collateral posted will presumably be released in the event that the underlying policy is not triggered, it will subsequently become available to pay claimants whose policy limits were not fully secured at inception. In our framework, this approach to collateralizing risk transfer offers the potential for welfare improvement relative to catastrophe bonds because of this increase in the availability of assets to pay claims.

In principle, varying the degree of collateralization across policies could be used to affect the allocation of assets during bankruptcy, but it is important to note the theoretical limits.

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<sup>17</sup>For more details, especially with respect to letters of credit, see Hall [15] and the NAIC’s *Receivers Handbook for Insurance Company Insolvencies*.

<sup>18</sup>Murray, A. (2006), “Reinsurance Side-Cars: Going Along for the Ride,” *Special Comment*, Moody’s Investor Service.

Varying the degree of collateralization associated with policies will generally give the insurer only limited control over the allocation of assets in bankruptcy. With multiple consumers, there are multiple bankruptcy states, with different consumers affected in different bankruptcy states: Varying ex ante collateral levels does not allow arbitrary prioritization of claims within bankruptcy.

Practical limits also apply. The effective security of partial collateralization will not be transparent to the policyholder in practice: To form an expectation of relative priority in bankruptcy, one must know all the details about the collateralization of other policies. For example, if all outstanding policies are collateralized to the same degree, collateralization has no effect, and recoveries would not change if the collateralization were terminated. Finally, in a world where claims are being submitted in continuous time, the reinsurer will not be able to commit all of its assets to ex ante collateralization, since it would have no funds available beyond the collateral supporting any given policy to pay a claim on that policy.

## 6.2 Third-Party Default Insurance

In the model, catastrophe bonds protect consumers from the consequences of insurer default. An alternative approach would be to buy “default insurance” from a third party. For example, credit default swaps (CDS) referencing outstanding transferable bonds or loans of the insurer<sup>19</sup> are a possible alternative hedging device: Issuing policies along with default protection could conceivably offer welfare improvements over the strategies considered in this paper. There are two aspects of this approach that merit comment.

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<sup>19</sup>Alternatively, in the absence of a CDS market, hedging strategies using equities or equity derivatives could substitute. However, equity-based strategies will generally have more basis risk as defined below.

First, to deliver a welfare improvement over catastrophe bonds, any protection offered by the CDS would have to be implicitly collateralized at less than 100% (or somehow have lower frictional costs with a similar degree of collateralization). On the one hand, less than full collateralization implies that there is some risk of counterparty default, so purchasing default insurance with a CDS is not a perfect substitute for the catastrophe bond. On the other hand, the seller of protection could conceivably realize additional economies by taking advantage of diversification opportunities beyond the insurance market.

Second, the CDS hedging approach will generally involve basis risk when there are more than two policyholders. With two policyholders, the CDS offers a perfect hedge to a consumer desiring protection: The contract is triggered only in states where the company defaults *and* the consumer experiences a loss. With multiple consumers, however, the company may default in states where the consumer does not experience a loss, thereby triggering a default insurance payment in a scenario where the consumer does not need additional indemnification. Moreover, the extent of a consumer's recovery on a policy will generally vary across states of default in the multiple consumer model, and this variation may not generally be hedged with a CDS (or with a short position in the insurer's debt, if this were possible) *unless* the holder of the underlying debt security is in the same class as the holder of an insurance policy with respect to priority of claim on the insurer's assets.<sup>20</sup>

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<sup>20</sup>This equivalence in priority will often hold for reinsurance contracts, which are in the same class as general creditors under the NAIC's Insurer Rehabilitation and Liquidation Model Act (revision of 4/27/04) but not for primary insurance contracts, which are typically assigned higher priority. In some circumstances, however, reinsurance contracts have also been assigned higher priority than general creditors (see Hall [14]).

## 7 Concluding Remarks

Catastrophe bonds fill an important niche in the risk transfer market. Contracting constraints prevent reinsurers from allocating assets efficiently in the event of insolvency. Catastrophe bonds and other forms of securitization effectively secure the claims of parties desiring more coverage in states where reinsurance counterparties are at risk of insolvency. Securitization as currently practiced thus has a well-defined economic role in the risk transfer market.

Some expect more. The success of asset-backed securitization—especially mortgage securitization—fueled predictions of a similar revolution in catastrophe insurance. And, although the revolution has yet to materialize, optimists cite the surge in catastrophe securitization after Hurricane Katrina, as well as the gradual development of mortgage securitization<sup>21</sup> in the 1970's, as evidence that revolution may still be in the offing. Is the catastrophe bond the next mortgage-backed security?

Our analysis suggests problems with the analogy. If securitization conquers the catastrophe risk transfer market, the conquest will be supported on fundamentals different from those in the mortgage market. The catastrophe bond's current reliance on full collateralization serves as an impediment to deep market penetration, one that can only be overcome by substantial frictional cost advantages, far beyond those observed in today's marketplace.

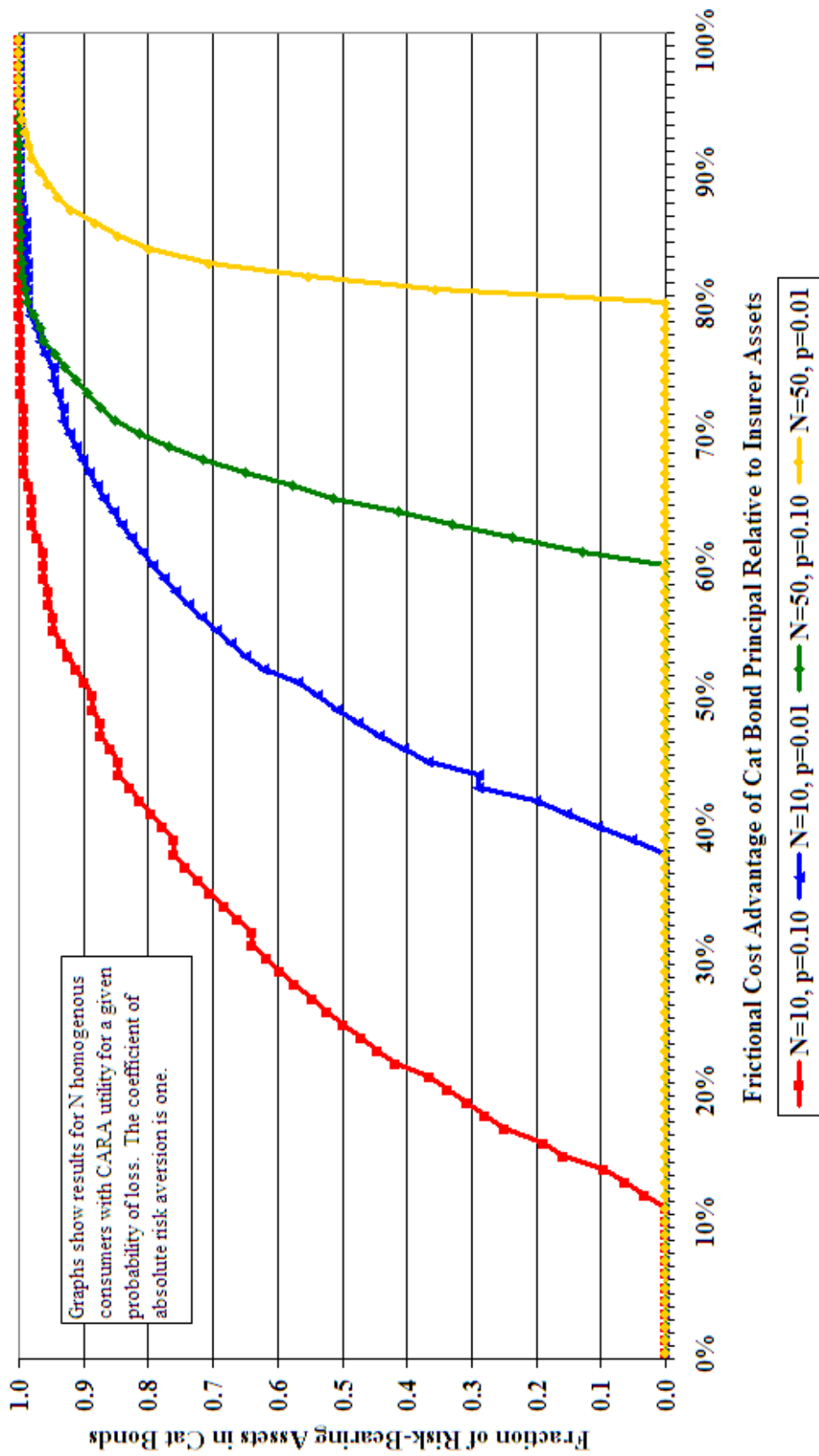
On the other hand, it is hard to know what the future will bring. Perhaps innovation in securitization technology will address the overcollateralization problem: Similar problems in

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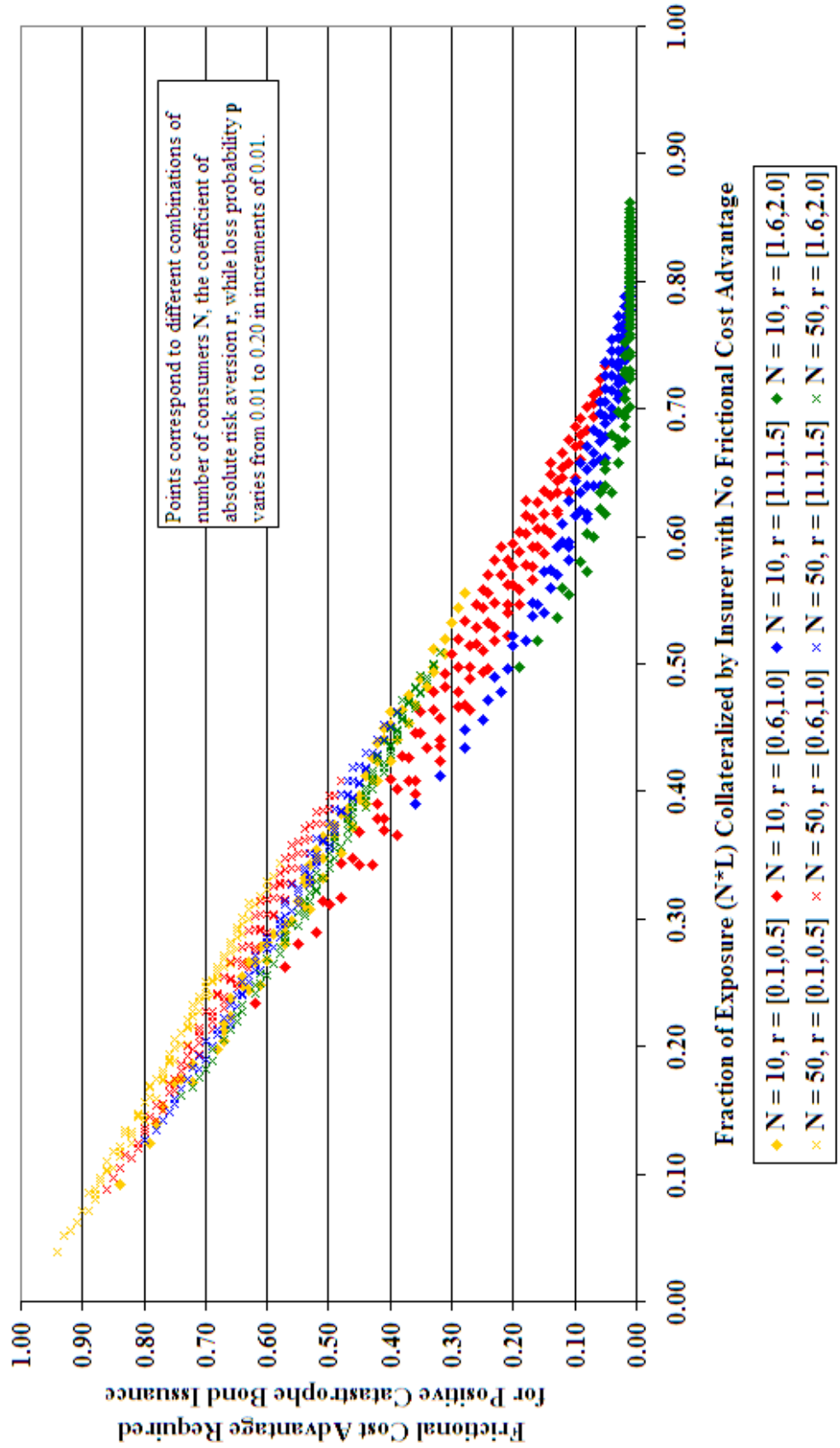
<sup>21</sup>According to the Federal Reserve's *Flow of Funds of the United States*, about 11% of single-family residential mortgages (in value terms) had been securitized by the end of 1980. This figure rose above 50% in the mid-1990's.

the secondary mortgage market of the 1970's were eventually solved (see Ranieri [25]). Even with fully collateralized structures, it is possible that significant frictional cost advantages will develop, in light of investor appetite for securities with limited investment horizons and triggers that guard against managerial discretion at reinsurance companies. As frictional cost advantages emerge, catastrophe securitization may become more attractive in segments of the market where diversification opportunities are limited. However, in arenas where significant diversification possibilities exist, catastrophe bonds must develop considerable frictional cost advantages before they can compete with traditional reinsurance. Indeed, the virtue of the catastrophe bond—its fixed and certain payout to a particular insured—flows from its greatest vice—the high cost of full collateralization.

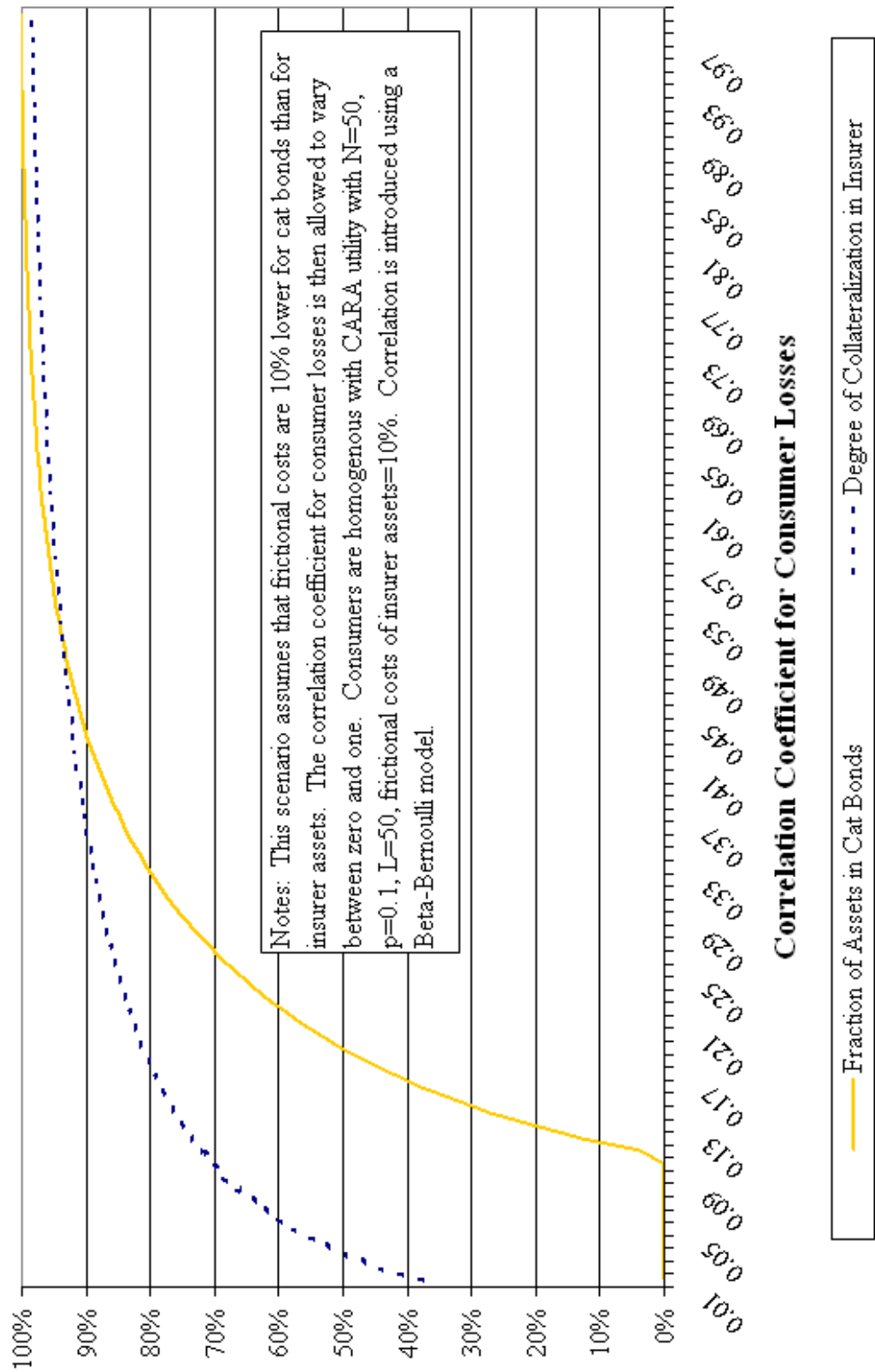
**Figure 1**  
**Catastrophe Bond Issuance as a Function of Frictional Cost Advantage**



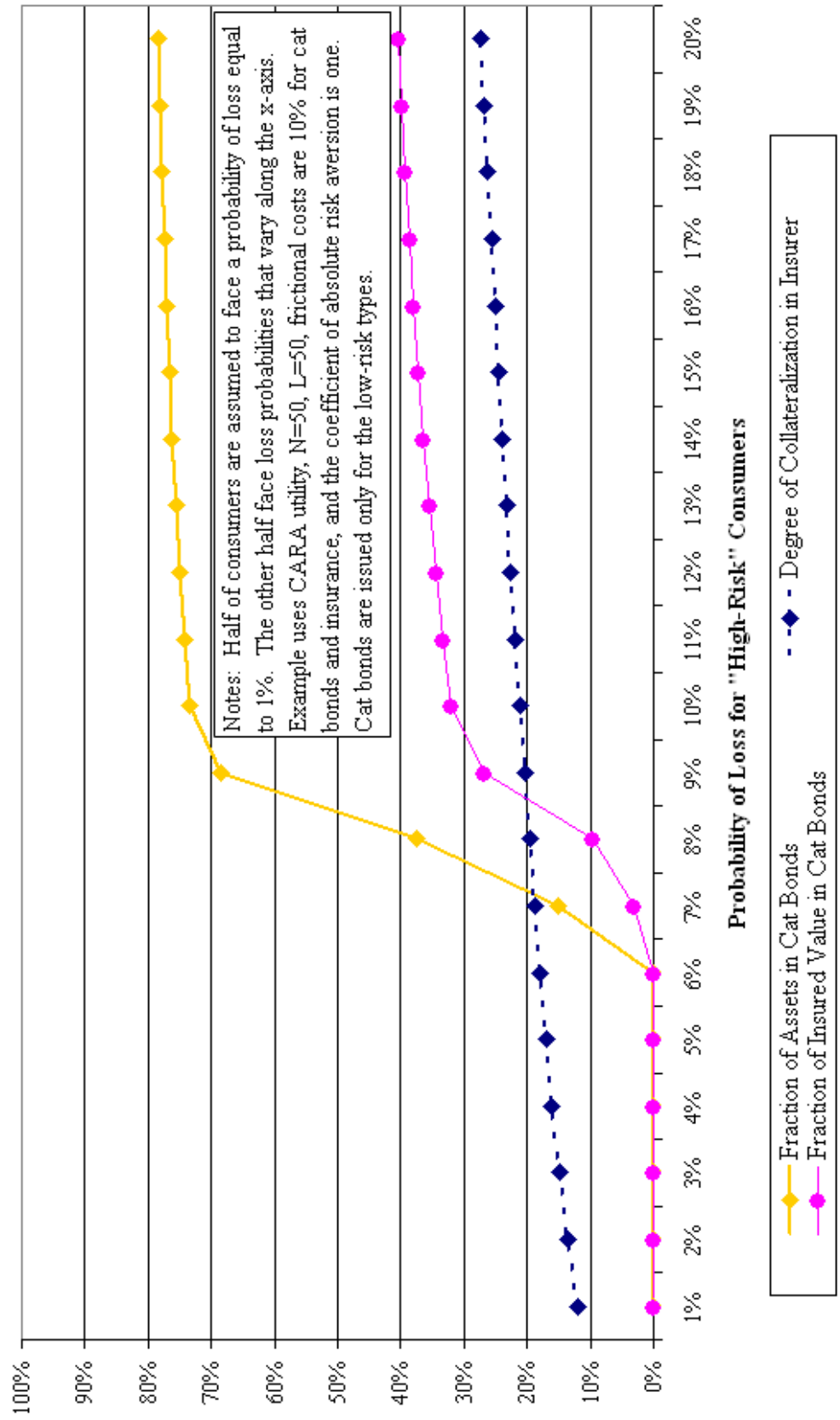
**Figure 2**  
**Frictional Cost Advantage Required for Catastrophe Bond Issuance As a**  
**Function of Collateralization in Insurance Market**



**Figure 3**  
**Correlated losses and Cat Bond Utilization**  
 with 10% Frictional Cost Advantage for Cat Bonds



**Figure 4**  
**Heterogeneous Loss Probabilities and Cat Bond Utilization**



# APPENDIX

## A Unconstrained Contracting

In this section of the Appendix, we verify that catastrophe bond issuance is optimally zero when insurance policy indemnity schedules can be made state-contingent and frictional costs for the two risk transfer technologies are equal.

Let  $I_i^{\mathbf{x}}$  represent the indemnity promised in state  $\mathbf{x}$  under the policy issued to consumer  $i$ . Then the utility of consumer  $i$  can be rewritten as:

$$EU_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i(W_i - L_i + f_{\mathbf{x}} I_i^{\mathbf{x}} + B_i - c_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i(W_i - c_i), \quad (14)$$

and the social planning problem becomes:

$$\max_{A, \{B_i\}, \{c_i\}, \{I_i^{\mathbf{x}}\}, \{f_{\mathbf{x}}\}} V = \sum_i EU_i \quad (15)$$

subject to:

$$[\mu] : \sum c_i \geq \delta_A A + \delta_B \sum_i B_i + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} f_{\mathbf{x}} I_i^{\mathbf{x}} + \sum_{i \in \Gamma(\mathbf{x})} B_i \right) \quad (16)$$

$$[\lambda_{\mathbf{x}}] : f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i^{\mathbf{x}} \leq A, \forall \mathbf{x} \quad (17)$$

$$[\phi_{\mathbf{x}}] : 0 \leq f_{\mathbf{x}} \leq 1, \forall \mathbf{x} \quad (18)$$

and the usual non-negativity constraints and subject to  $I_i^{\mathbf{x}} = 0$  for  $\mathbf{x} \notin \Omega^i$ , for all  $i$ . The following theorem proves the result.

**Theorem.** *Suppose  $\delta_A = \delta_B = \delta$ . Suppose  $\hat{A}, \{\hat{B}_i\}, \{\hat{c}_i\}, \{\hat{I}_i^{\mathbf{x}}\}, \{\hat{f}_{\mathbf{x}}\}$  is a set of optimal choices maximizing (15) subject to the listed constraints, with  $\hat{B}_k > 0$  for at least one  $k \in \{1, \dots, N\}$ . Then there exists another set of choices  $\bar{A}, \{\bar{B}_i\}, \{\bar{c}_i\}, \{\bar{I}_i^{\mathbf{x}}\}, \{\bar{f}_{\mathbf{x}}\}$ , with  $\bar{B}_i = 0$  for all  $i$ , that satisfy the listed constraints and also maximize the objective function.*

**Proof:**

Define  $\Lambda = \{k : k \in \{1, \dots, N\}, \hat{B}_k > 0\}$ .

Let  $\bar{c}_i = \hat{c}_i$ , for all  $i$ .

Let  $\bar{B}_i = 0$ , for all  $i$ .

Let  $\bar{f}_{\mathbf{x}} = 1$ , for all  $\mathbf{x}$ .

Let  $\bar{A} = \hat{A} + \sum_i \hat{B}_i$

For all  $i$ : Let  $\bar{I}_i^{\mathbf{x}} = \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \hat{B}_i, \forall \mathbf{x} \in \Omega^i; \bar{I}_i^{\mathbf{x}} = \hat{I}_i^{\mathbf{x}} = 0, \forall \mathbf{x} \notin \Omega^i$ .

We first verify that the new choices satisfy all constraints:

Note that

$$\begin{aligned}
\sum \bar{c}_i &= \sum \hat{c}_i \geq \delta \left( \bar{A} + \sum_i \bar{B}_i \right) + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} \bar{f}_{\mathbf{x}} \bar{I}_i^{\mathbf{x}} + \sum_{i \in \Gamma(\mathbf{x})} \bar{B}_i \right) \\
&= \delta \left( \hat{A} + \sum_i \hat{B}_i \right) + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} \left( \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \hat{B}_i \right) \right) \\
&= \delta \left( \hat{A} + \sum_i \hat{B}_i \right) + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \sum_{i \in \Gamma(\mathbf{x})} \hat{B}_i \right),
\end{aligned}$$

so  $[\mu]$  is satisfied.

For all  $\mathbf{x}$ ,

$$\bar{f}_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} \bar{I}_i^{\mathbf{x}} = \sum_{i \in \Gamma(\mathbf{x})} \left( \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \hat{B}_i \right)$$

But

$$\bar{A} = \hat{A} + \sum_i \hat{B}_i \geq \sum_{i \in \Gamma(\mathbf{x})} \left( \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \hat{B}_i \right)$$

since

$$\hat{A} \geq \sum_{i \in \Gamma(\mathbf{x})} \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}},$$

so  $[\lambda_{\mathbf{x}}]$  is satisfied.

Since  $\bar{f}_{\mathbf{x}} = 1$ , for all  $\mathbf{x}$ ,  $[\phi_{\mathbf{x}}]$  is satisfied, and the non-negativity constraints are obviously satisfied since the original set of choices met the non-negativity constraints. So the proposed choices are feasible ones.

Now we show that the proposed alternative choices produce the same value of the objective function. Note that

$$EU_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i(W_i - L_i + \bar{f}_{\mathbf{x}} \bar{I}_i^{\mathbf{x}} + \bar{B}_i - \bar{c}_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i(W_i - \bar{c}_i)$$

can be rewritten as:

$$\sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i(W_i - L_i + \hat{f}_{\mathbf{x}} \hat{I}_i^{\mathbf{x}} + \hat{B}_i - \hat{c}_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i(W_i - \hat{c}_i),$$

so, for each consumer, the proposed alternative choices yield the exact same utility as the original choices. Therefore, the objective function value does not change when we move from the original choices to the new choices. Q.E.D.

## B Proofs for Section 4

### B.1 Derivation of (13)

We start by developing the first order conditions from the stated maximization problem. We then use these to derive Equation 13. For notational ease, define the following terms:

$$\begin{aligned} L(\mathbf{x}) &\equiv \sum_{i \in \Gamma(\mathbf{x})} I_i \\ S(\mathbf{x}) &\equiv A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \\ \sigma_{\mathbf{x}} &\equiv \lambda_{\mathbf{x}} - (1 - f_{\mathbf{x}})\Lambda_{\mathbf{x}} \end{aligned}$$

$L(x)$  represents the insurance company's total liabilities in state  $x$ ;  $S(x)$  is total company surplus in state  $x$ ; and  $\sigma_x$  is the value of assets in state  $x$ . The first order conditions are as follows (where we use the notation  $U_i^{\mathbf{x}}$  to denote the utility of consumer  $i$  in state  $\mathbf{x}$ ):

$$[B_i] : \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \left( \delta_B + \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) \right) \leq 0 \quad (19)$$

$$[c_i] : - \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \frac{\partial U_i^{\mathbf{x}}}{\partial W} + \mu = 0 \quad (20)$$

$$[I_i] : \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) f_{\mathbf{x}} \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{\mathbf{x} \in \Omega^i} f_{\mathbf{x}} \sigma_{\mathbf{x}} = 0 \quad (21)$$

$$[A] : -\delta_A \mu + \sum_{\mathbf{x} \in \Omega} \sigma_{\mathbf{x}} = 0 \quad (22)$$

$$[f_{\mathbf{x}}] : \sum_{i \in \Gamma(\mathbf{x})} \Pr(\mathbf{x}) I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sigma_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i - \phi_{\mathbf{x}} + \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) = 0 \quad (23)$$

After multiplying by  $f_{\mathbf{x}}$ , and rearranging, note that  $[f_{\mathbf{x}}]$  can be written as:

$$\sum_{i \in \Gamma(\mathbf{x})} \Pr(\mathbf{x}) w_i^{\mathbf{x}} f_{\mathbf{x}} \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - f_{\mathbf{x}} \sigma_{\mathbf{x}} - \frac{\phi_{\mathbf{x}} f_{\mathbf{x}}}{L(\mathbf{x})} + \frac{\Lambda_{\mathbf{x}} f_{\mathbf{x}} S(\mathbf{x})}{L(\mathbf{x})} = 0,$$

where

$$w_i^{\mathbf{x}} = \frac{I_i}{L(\mathbf{x})}.$$

Summing over  $\mathbf{x} \in \Omega^i$ :

$$\sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) f_{\mathbf{x}} \left( \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{\mathbf{x} \in \Omega^i} f_{\mathbf{x}} \sigma_{\mathbf{x}} - \sum_{\mathbf{x} \in \Omega^i} \frac{f_{\mathbf{x}} \phi_{\mathbf{x}}}{L(\mathbf{x})} + \sum_{\mathbf{x} \in \Omega^i} \frac{\Lambda_{\mathbf{x}} f_{\mathbf{x}} S(\mathbf{x})}{L(\mathbf{x})} = 0.$$

In summary, we have

$$\sum_{\mathbf{x} \in \Omega^i} f_{\mathbf{x}} \sigma_{\mathbf{x}} = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) f_{\mathbf{x}} \left( \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{\mathbf{x} \in \Omega^i} \frac{f_{\mathbf{x}} \phi_{\mathbf{x}}}{L(\mathbf{x})} + \sum_{\mathbf{x} \in \Omega^i} \frac{\Lambda_{\mathbf{x}} f_{\mathbf{x}} S(\mathbf{x})}{L(\mathbf{x})}. \quad (24)$$

Note that we did not need to multiply by  $f_{\mathbf{x}}$ . Omitting this step leads to:

$$\sum_{\mathbf{x} \in \Omega^i} \sigma_{\mathbf{x}} = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) \left( \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{\mathbf{x} \in \Omega^i} \frac{\phi_{\mathbf{x}}}{L(\mathbf{x})} + \sum_{\mathbf{x} \in \Omega^i} \frac{\Lambda_{\mathbf{x}} S(\mathbf{x})}{L(\mathbf{x})} \quad (25)$$

Now recall the marginal condition  $[B_i]$ :

$$R_i = -(\delta_B + \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}))\mu + \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) \frac{\partial U_i^{\mathbf{x}}}{\partial W} \leq 0.$$

The first term is the marginal cost of issuance—including both the frictional cost per dollar of collateral and the expected loss on the bond—and the second term is the marginal benefit, which amounts to an extra dollar of consumption in all of the loss states. Subtracting the left-hand side of the first order condition  $[I_i]$  from the above expression yields the following:

$$R_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) + \sum_{\mathbf{x} \in \Omega^i} f_{\mathbf{x}} \sigma_{\mathbf{x}} - \delta_B \mu.$$

or

$$R_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) + \sum_{\mathbf{x} \in \Omega^i} [f_{\mathbf{x}} - 1] \sigma_{\mathbf{x}} + \sum_{\mathbf{x} \in \Omega^i} \sigma_{\mathbf{x}} - \delta_B \mu.$$

Substituting in from  $[A]$  yields:

$$R_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) + \sum_{\mathbf{x} \in \Omega^i} [f_{\mathbf{x}} - 1] \sigma_{\mathbf{x}} - \sum_{\mathbf{x} \notin \Omega^i} \sigma_{\mathbf{x}} + (\delta_A - \delta_B) \mu.$$

Subtracting (25) from (24) and substituting in the resulting expression for  $\sum_{\mathbf{x} \in \Omega^i} [f_{\mathbf{x}} - 1] \sigma_{\mathbf{x}}$  yields

$$R_i = \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) [1 - f_{\mathbf{x}}] \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \sum_{j \in \Gamma(\mathbf{x})} w_j^{\mathbf{x}} \frac{\partial U_j^{\mathbf{x}}}{\partial W} \right) - \sum_{\mathbf{x} \notin \Omega^i} \sigma_{\mathbf{x}} - (\delta_A - \delta_B) \mu + (1 - f_{\mathbf{x}}) \left[ \sum_{\mathbf{x} \in \Omega^i} \frac{\phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} S(\mathbf{x})}{L(\mathbf{x})} \right] \leq 0$$

Finally, observe that the problem's Kuhn-Tucker conditions imply that  $\phi_{\mathbf{x}}(1 - f_{\mathbf{x}})$  is zero for all values of  $\mathbf{x}$ , as is  $\Lambda_{\mathbf{x}}(1 - f_{\mathbf{x}})S(\mathbf{x})$ . This implies that the last term of the above expression is zero. The result is then equation 13.

## B.2 Irrelevance of Catastrophe Bonds in the Absence of Default Risk

In Footnote 12, we claim that a solution with positive catastrophe bond issuance for consumer  $i$ , where that consumer faces no default risk, is not unique and can be replicated by a solution with zero issuance for consumer  $i$ . The following theorem proves this claim formally.

**Theorem.** *Suppose  $\delta_A = \delta_B = \delta$  and that  $\hat{A}, \{\hat{B}_i\}, \{\hat{c}_i\}, \{\hat{I}_i\}, \{\hat{f}_x\}$  is a set of optimal choices maximizing (8) subject to the listed constraints, with  $\hat{B}_k > 0$  for some consumer  $k \in \{1, \dots, N\}$ , and  $\hat{f}_x = 1$  for all  $\mathbf{x} \in \Omega^k$ . Then there exists another set of choices  $\bar{A}, \{\bar{B}_i\}, \{\bar{c}_i\}, \{\bar{I}_i\}, \{\bar{f}_x\}$ , with  $\bar{B}_k = 0$  that satisfy the listed constraints and also maximize the objective function.*

**Proof:**

Let  $\bar{c}_i = \hat{c}_i$ , for all  $i$ .

Let  $\bar{B}_i = \hat{B}_i$ , for all  $i \neq k$ . Let  $\bar{B}_k = 0$ .

Let  $\bar{I}_i = \hat{I}_i$ , for all  $i \neq k$ . Let  $\bar{I}_k = \hat{I}_k + \hat{B}_k$

Let  $\bar{f}_x = \hat{f}_x$ , for all  $\mathbf{x}$ .

Let  $\bar{A} = \hat{A} + \hat{B}_k$

We first verify that all constraints are satisfied. Note that

$$\begin{aligned} \sum \bar{c}_i &= \sum \hat{c}_i \geq \delta \left( \hat{A} + \sum_i \hat{B}_i \right) + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} \hat{f}_x \hat{I}_i + \sum_{i \in \Gamma(\mathbf{x})} \hat{B}_i \right) = \\ &\delta \left( \bar{A} + \sum_i \bar{B}_i \right) + \sum_{\mathbf{x} \in \Omega} \Pr(\mathbf{x}) \left( \sum_{i \in \Gamma(\mathbf{x})} \bar{f}_x \bar{I}_i + \sum_{i \in \Gamma(\mathbf{x})} \bar{B}_i \right), \end{aligned}$$

so  $[\mu]$  is satisfied.

For all  $\mathbf{x} \notin \Omega^k$ ,  $[\lambda_x]$  is obviously satisfied. For  $\mathbf{x} \in \Omega^k$ , we know that

$$\sum_{i \in \Gamma(\mathbf{x})} \bar{I}_i = \sum_{i \in \Gamma(\mathbf{x})} \hat{I}_i + \hat{B}_k,$$

and that  $\bar{f}_x = \hat{f}_x = 1$  for all  $\mathbf{x} \in \Omega^k$ . So, it follows that

$$\bar{f}_x \sum_{i \in \Gamma(\mathbf{x})} \bar{I}_i = \sum_{i \in \Gamma(\mathbf{x})} \hat{I}_i + \hat{B}_k \leq \hat{A} + \hat{B}_k = \bar{A},$$

so  $[\lambda_x]$  is also satisfied for  $\mathbf{x} \in \Omega^k$ .

Finally, the constraints  $[\phi_x]$  and  $[\Lambda_x]$  are obviously satisfied, since  $\bar{f}_x = \hat{f}_x = 1$ , for all  $\mathbf{x}$ .

Next, we verify that the objective function value is unchanged with the new choices.

Note that, for all  $i$ ,

$$\sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i (W_i - L_i + \bar{f}_x \bar{I}_i + \bar{B}_i - \bar{c}_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i (W_i - \bar{c}_i)$$

equals

$$\sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) U_i (W_i - L_i + \hat{f}_x \hat{I}_i + \hat{B}_i - \hat{c}_i) + \sum_{\mathbf{x} \notin \Omega^i} \Pr(\mathbf{x}) U_i (W_i - \hat{c}_i).$$

Thus, expected utility for each consumer under the new choices is identical to that under the original choices. It follows that the overall objective function  $V = \sum EU_i$  must have the same value under each set of choices. Q.E.D.

### B.3 Homogeneity with Identical Frictional Costs

In this section, we assume that consumers are “homogenous” in the sense of having identical preferences and wealth, as well as having risks that are independent and identically distributed. In this homogeneous case, we restrict ourselves to considering symmetric equilibria in which all consumers are treated identically. With this focus, the choice problem can be simplified by recognizing that all consumers have the same contracts and bond issuance.

**Lemma.** *Under homogeneity, the optimum features  $\phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) = 0$  for each  $\mathbf{x}$ .*

**Proof:** Start by multiplying  $[I_i]$  (see Section B.1) by  $I_i$  and summing over  $i$  to yield

$$\sum_i \sum_{\mathbf{x} \in \Omega^i} \Pr(\mathbf{x}) f_{\mathbf{x}} I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_i \sum_{\mathbf{x} \in \Omega^i} f_{\mathbf{x}} \sigma_{\mathbf{x}} I_i = 0.$$

Next, multiply  $[f_{\mathbf{x}}]$  by  $f_{\mathbf{x}}$  and sum over  $\mathbf{x}$  to yield:

$$\sum_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} \Pr(\mathbf{x}) f_{\mathbf{x}} I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} f_{\mathbf{x}} \sigma_{\mathbf{x}} I_i - \sum_{\mathbf{x}} f_{\mathbf{x}} \left[ \phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) \right] = 0 \quad (26)$$

After noting that, for  $\mathbf{x} \notin \cup_{i=1}^N \Omega^i$ , each of the three summation terms in (26) is zero, we conclude that the foregoing implies that:

$$\sum_{\mathbf{x}} f_{\mathbf{x}} \left[ \phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) \right] = 0. \quad (27)$$

Now divide the states of the world into two groups—those for which  $f_{\mathbf{x}} = 1$  and those for which  $f_{\mathbf{x}} < 1$ . Consider the latter group first.

For  $f_{\mathbf{x}} < 1$ , multiply  $[f_{\mathbf{x}}]$  by  $(1 - f_{\mathbf{x}})$  to yield:

$$\sum_{i \in \Gamma(\mathbf{x})} \Pr(\mathbf{x}) (1 - f_{\mathbf{x}}) I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{i \in \Gamma(\mathbf{x})} (1 - f_{\mathbf{x}}) \sigma_{\mathbf{x}} I_i - \sum_{\mathbf{x}} (1 - f_{\mathbf{x}}) \left[ \phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) \right]$$

The Kuhn-Tucker conditions imply that the last summation term is zero. But this further implies, since  $f_{\mathbf{x}} < 1$ , that:

$$\sum_{i \in \Gamma(\mathbf{x})} \Pr(\mathbf{x}) I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \sum_{i \in \Gamma(\mathbf{x})} \sigma_{\mathbf{x}} I_i = 0,$$

which in turn implies that,

$$\phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) = 0 \quad \forall \mathbf{x} \mid f_{\mathbf{x}} < 1. \quad (28)$$

For  $f_{\mathbf{x}} = 1$ , if such state(s) exists, note that homogeneity implies that  $\frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu$  will have the same value for all  $i$  and all  $\mathbf{x}$  satisfying the assumed equality. Furthermore, (27) and (28) imply that:

$$\sum_{\mathbf{x} \mid f_{\mathbf{x}}=1} \left[ \phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) \right] = 0 \quad (29)$$

Note further that  $\sigma_{\mathbf{x}} = 0$  when  $f_{\mathbf{x}} = 1$ . This is obviously true for any  $\mathbf{x}$  when  $\sum_{i \in \Gamma(\mathbf{x})} I_i < A$ . Moreover, if  $\sum_{i \in \Gamma(\mathbf{x})} I_i = A$  for some  $\mathbf{x}$ , and  $\sigma_{\mathbf{x}}$  is non-zero, then  $[f_{\mathbf{x}}]$  reduces to:

$$\sum_{i \in \Gamma(\mathbf{x})} \text{Pr}(\mathbf{x}) I_i \left( \frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu \right) - \lambda_{\mathbf{x}} - \phi_{\mathbf{x}} = 0,$$

which implies that  $\frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu > 0$  and, consequently, a violation of (29).

Therefore, it follows that:

$$\frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu = 0 \quad \forall \mathbf{x} \mid f_{\mathbf{x}} = 1$$

and

$$\phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) = 0 \quad \forall \mathbf{x} \mid f_{\mathbf{x}} = 1.$$

Q.E.D.

Based on the foregoing analysis, and making use of the results that  $\phi_{\mathbf{x}} - \Lambda_{\mathbf{x}} \left( A - f_{\mathbf{x}} \sum_{i \in \Gamma(\mathbf{x})} I_i \right) = 0$  for all  $\mathbf{x}$ , and  $\frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu = 0 \quad \forall \mathbf{x} \mid f_{\mathbf{x}} = 1$ , we can now make the following characterizations based on the marginal condition  $[f_{\mathbf{x}}]$ :

1.  $\sigma_{\mathbf{x}} = 0$  when  $f_{\mathbf{x}} = 1$
2.  $\sigma_{\mathbf{x}} > 0$ ,  $\frac{\partial U_i^{\mathbf{x}}}{\partial W} - \mu > 0$  when  $f_{\mathbf{x}} < 1$

Now, as noted in the text, the marginal condition for catastrophe bond issuance under homogeneity and identical frictional costs reduces to:

$$R_i = - \sum_{\mathbf{x} \notin \Omega^i} \sigma_{\mathbf{x}} \leq 0.$$

This condition is evidently strictly less than zero if  $f_{\mathbf{x}} < 1$  for any  $\mathbf{x} \notin \Omega^i$ . In other words, for catastrophe bond issuance will be strictly suboptimal unless consumers are fully insured in

all states of the world *except* the one where *everyone* experiences a loss. In this scenario, the social planner is indifferent between using reinsurance policies alone and reinsurance policies in conjunction with catastrophe bonds.

## C Optimal Collateralization with Generalized Capital Costs

### C.1 Theoretical Development

Little evidence exists connecting the cost of capital in the insurance industry to the risk characteristics of the underlying policyholder liabilities.<sup>22</sup> Nevertheless, a theoretical connection exists—so we explore the possibility in this subsection by allowing the cost of risk transfer to depend on the relationship between the risk transferred and the risks in the broader capital markets.

To add security markets and equilibrium to our model, we start with a state-pricing approach built on the assumption of no arbitrage, as described in the first chapter of Duffie [10]. For insurance markets to be relevant (and not redundant), it must be the case that financial markets are incomplete. Specifically, if it is possible to replicate the payoffs from an insurance policy using existing securities in frictionless markets (and if those securities are priced fairly), there will be no need for insurance policies. Therefore, we will work under the assumption that markets are incomplete—or, specifically, that insurance policy payoffs cannot be replicated using other financial instruments. This is similar in flavor to the assumption underlying Mayers and Smith [21]—tradeable portfolio securities are assumed to have well-defined prices that flow from an equilibrium asset pricing model, but insurance risks are denoted as “nontradeable” and thus must be dealt with separately (although, as noted by Mayers and Smith, the decisions are not independent).

A key question not addressed by Mayers and Smith, but crucial for our purpose, is how insurance policies are priced when they are non-tradeable and cannot be replicated with other financial instruments. The obvious approach is to price insurance policies “as if” they were traded financial securities. That is, their prices are determined by their contingent payoffs, weighted by appropriate state prices—just as with any other security. The only complication is that, by assumption, the contingencies relevant for insurance policy payoffs do not map into the state space that governs the security markets—so the state prices needed to price policies will not follow from the absence of arbitrage among the financial instruments traded in the security markets. With this in mind, we extend state prices derived from the assumption of no-arbitrage in the security markets to apply to subsets of events within states. While this extension is not a technical implication of arbitrage pricing theory, it yields a foundation for insurance pricing that is logically consistent with security market pricing.

We use the  $N$  person model and start by defining the relevant probability spaces. Recall the set of vectors that define consumer loss experience, denoted by  $\Omega$ , whose members are

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<sup>22</sup>See Cummins and Harrington [6] and Cox and Rudd [5] for studies of the connection between returns on insurance liability portfolios and stock market returns. Hoyt and McCullough [16] and Litzenberger, Beaglehole, and Reynolds [19] study catastrophe losses in particular, finding them to be uncorrelated with stock market returns.

row vectors  $\mathbf{x}$  of length  $N$ , with the vector elements all taking a value of zero or one:  $\mathbf{x}(i) = 1$  means that consumer  $i$  experienced a loss, while  $\mathbf{x}(i) = 0$  means that she did not.

We now introduce a set of  $M$  securities, each security with distinct payoffs in  $S$  states of the world. Let  $\Psi$  be the set of those states (with the associated  $\sigma$ -algebra  $\mathcal{F}_\Psi$ ), and define state prices, consistent with the absence of arbitrage, denoted by  $\pi_s$  for each  $s \in \Psi$ . Let  $D$  be an  $M \times S$  matrix, with  $D_{ij}$  describing the payoff of the  $i$ -th security in the  $j$ -th state. We assume that

$$\text{span}(D) \equiv \mathbb{R}^S.$$

This condition is typically known as a “complete markets” condition—that any arbitrary menu of state-contingent consumption can be purchased at time zero. In our case, however, it would be misleading to characterize markets as complete, since  $\Psi$  does not provide a complete description of the states of the world.

Instead, we characterize the full probability space as  $(\Theta, \mathcal{F}_\Theta, \omega_\Theta)$ , with

$$\Theta \equiv \{\boldsymbol{\theta} = [\mathbf{x}(1) \ \mathbf{x}(2) \dots \mathbf{x}(N) \ s] \mid \mathbf{x} \in \Omega, s \in \Psi\}$$

The state variable  $\boldsymbol{\theta} \in \Theta$  is a row vector of length  $N + 1$  that provides a complete description of one possible state of the world. The first  $N$  elements of  $\boldsymbol{\theta}$  describe which consumers experienced losses (and which ones did not), while the last element describes the state of the securities markets. The entire set  $\Theta$  contains all possible states of the world.

The following set definitions are useful:

$$\Theta^i = \{\boldsymbol{\theta} : \boldsymbol{\theta}(i) = 1\},$$

the set of all states in which agent  $i$  suffers a loss, and

$$\Gamma(\boldsymbol{\theta}) = \{i : \boldsymbol{\theta}(i) = 1\},$$

the set of all agents that lose in state  $\boldsymbol{\theta}$ . In addition, for every  $s$  and every agent  $i$ , define:

$$\Upsilon_i^{s0} \equiv \{\boldsymbol{\theta} : \boldsymbol{\theta} \notin \Theta^i, \boldsymbol{\theta}(N + 1) = s\},$$

the set of all states  $\boldsymbol{\theta}$  where agent  $i$  does not suffer a loss and the security market “sub-state” is  $s$ , and

$$\Upsilon_i^{s1} \equiv \{\boldsymbol{\theta} : \boldsymbol{\theta} \in \Theta^i, \boldsymbol{\theta}(N + 1) = s\},$$

the set of all states  $\boldsymbol{\theta}$  where agent  $i$  *does* suffer a loss and the security market “sub-state” is  $s$ . Finally, note that for any  $i$  the entire sub-state space defined by  $s$  can be written as:

$$\Upsilon^s \equiv \Upsilon_i^{s1} \cup \Upsilon_i^{s0}$$

We now extend the state prices to define prices for events that are not measurable with respect to  $\mathcal{F}_\Psi$ . Define extended state prices as follows. For each  $s \in \Psi$

$$\pi^\boldsymbol{\theta} = \pi_s, \forall \boldsymbol{\theta} \in \Theta \mid \boldsymbol{\theta}(N + 1) = s$$

Recall that we are extending the state-prices to include the price of claims associated with the hazards being insured. This approach implicitly assumes that there is no variation in “sub-state” prices within the states priced by the security market equilibrium. As noted earlier, the absence of arbitrage does not pin down the state prices for events that are not measurable with respect to  $\mathcal{F}_\Psi$ , but this assumption provides a basis for insurance pricing that is logically consistent with the security market equilibrium.

We can now define utility for Consumer  $i$  as

$$EU_i = \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_i^{s1}} \omega_\Theta(\theta) U_i(W_{is} - L_i + f_\theta I_i + B_i) + \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_i^{s0}} \omega_\Theta(\theta) U_i(W_{is}),$$

where  $f_\theta$  represents the proportion of the indemnity payment actually paid in state  $\theta$ , and  $W_{is}$  is state-contingent security-market wealth for consumer  $i$ .

The social planning problem can now be written as:

$$\max_{A, \{B_i\}, \{c_i\}, \{I_i\}, \{f_\theta\}, \{W_{is}\}} V = \sum_i EU_i \quad (30)$$

subject to

$$[\mu] : \sum c_i \geq \delta_A A + \delta_B \sum_i B_i + \sum_{\theta \in \Theta} \left( \omega_\Theta(\theta) \pi^\theta \sum_{i \in \Gamma(\theta)} (f_\theta I_i + B_i) \right) \quad (31)$$

$$[\lambda_\theta] : f_\theta \sum_{i \in \Gamma(\theta)} I_i \leq A, \forall \theta \quad (32)$$

$$[\phi_\theta] : f_\theta \leq 1, \forall \theta \quad (33)$$

$$[\Lambda_\theta] : (1 - f_\theta) \left( A - f_\theta \sum_{i \in \Gamma(\theta)} I_i \right) \leq 0 \quad (34)$$

$$[\varphi_i] : \sum_{s \in \Psi} \pi_s W_{is} \leq W - c_i, \forall i \quad (35)$$

and the previous non-negativity constraints. New to the problem is the last constraint, which governs portfolio investment. The present value of each consumer’s contingent consumption is constrained to no more than time zero wealth, net of risk transfer costs.

Before proceeding further, it is worth noting that the generalized capital allocation structure does not alter our basic results for the return on catastrophe bonds (see Appendix Section C.2). In this setting, the return to catastrophe bonds is given by:

$$R_i = \sum_{\theta \in \Theta^i} \omega_\Theta(\theta) (1 - f_\theta) \left( \frac{\partial U_i^\theta}{\partial W} - \sum_{j \in \Gamma(\theta)} w_j^\theta \frac{\partial U_j^\theta}{\partial W} \right) - \sum_{\theta \notin \Theta^i} \sigma_\theta + \mu(\delta_A - \delta_B) \leq 0 \quad (36)$$

This is analogous to Equation 13, with  $\frac{\partial U_i^\theta}{\partial W}$  is the marginal utility of wealth for consumer  $i$

in state  $\theta$ , and:

$$\begin{aligned} w_j^\theta &\equiv \frac{I_j}{\sum_{j \in \Gamma(\theta)} I_j}, \\ \sigma_\theta &\equiv \lambda_\theta - (1 - f_\theta)\Lambda_\theta. \end{aligned}$$

The marginal conditions describing optimal issuance of catastrophe bonds do not directly reference frictional costs (except when  $\delta_B \neq \delta_A$ ), yet frictional costs are required for them to be useful. Without frictional costs, both terms of the marginal condition will be zero in all cases—simply because there is no insurance company default in the absence of frictional costs, and every consumer enjoys full insurance.

To illustrate this, we eliminate frictional costs but continue to weight claims payments and catastrophe bond default according to the appropriate state prices:

$$\sum c_i \geq \sum_{\theta \in \Theta} \left( \omega_\Theta(\theta) \pi^\theta f_\theta \sum_{i \in \Gamma(\theta)} I_i \right) + \sum_i \left( \sum_{\theta \in \Theta^i} \omega_\Theta(\theta) \pi^\theta \right) B_i.$$

Intuitively, insurance premiums must pay for contingent claims on the assets, but, in the absence of frictional costs, extra capital always earns its own keep, because the insurer can invest it at market rates of return. As a result, the only real cost of holding capital as collateral is the extent to which you are exposing the owner to additional claims risk—but this additional risk is compensated in the capital market.

Section C.3 of the Appendix characterizes the solution in this setting without frictional costs in a series of lemmas. There are two important lessons. First, full insurance will always be optimal: Every consumer will be fully covered. Second, the optimal division of collateral between reinsurance company assets and catastrophe bond principal is indeterminate. In other words, with full insurance it ends up being irrelevant whether the insurance is provided through reinsurance policies or catastrophe bonds: Any combination of the two instruments is optimal, so long as full insurance is provided.

As is often the case, consumers fully insure when the insurance is fairly priced. Moreover, there is no incentive to economize on collateral in this setting—collateral is “free” in the sense that there are no frictional costs. The only “costs” with holding assets in the reinsurer or the SPV are the fair value of expected claims payments to policyholders or catastrophe bond issuers. Since there are no penalties associated with over-collateralization, neither risk transfer instrument holds a natural advantage over the other.

## C.2 Derivation of (36)

In what follows, we adapt the notation of Section 4 as follows:

$$\begin{aligned} S_\theta &\equiv A - f_\theta \sum_{i \in \Gamma(\theta)} I_i \\ \sigma_\theta &= \lambda_\theta - \Lambda_\theta(1 - f_\theta) \end{aligned}$$

The generalized asset allocation model produces the following first order conditions that analogize to the simpler model:

$$[B_i] : \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} \omega_{\Theta}(\boldsymbol{\theta}) \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \left( \delta_B + \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \right) \leq 0, \forall i \quad (37)$$

$$[c_i] : -\varphi_i + \mu = 0, \forall i \quad (38)$$

$$[I_i] : \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} \omega_{\Theta}(\boldsymbol{\theta}) f_{\boldsymbol{\theta}} \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \pi^{\boldsymbol{\theta}} \right) - \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} f_{\boldsymbol{\theta}} \sigma_{\boldsymbol{\theta}} = 0, \forall i \quad (39)$$

$$[A] : -\delta_A \mu + \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon^{\mathbf{s}}} \sigma_{\boldsymbol{\theta}} = 0 \quad (40)$$

$$[f_{\boldsymbol{\theta}}] : \sum_{i \in \Gamma(\boldsymbol{\theta})} \omega_{\Theta}(\boldsymbol{\theta}) I_i \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \pi^{\boldsymbol{\theta}} \right) - \sigma_{\boldsymbol{\theta}} \sum_{i \in \Gamma(\boldsymbol{\theta})} I_i - \phi_{\boldsymbol{\theta}} + \Lambda_{\boldsymbol{\theta}} S_{\boldsymbol{\theta}} = 0, \forall \boldsymbol{\theta} \quad (41)$$

Multiply  $[f_{\boldsymbol{\theta}}]$  by  $(f_{\boldsymbol{\theta}} - 1)$  and divide by  $\sum_{i \in \Gamma(\boldsymbol{\theta})} I_i$  to obtain:

$$\sum_{i \in \Gamma(\boldsymbol{\theta})} \omega_{\Theta}(\boldsymbol{\theta}) (f_{\boldsymbol{\theta}} - 1) w_i^{\boldsymbol{\theta}} \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \pi^{\boldsymbol{\theta}} \right) - \sigma_{\boldsymbol{\theta}} (f_{\boldsymbol{\theta}} - 1) - \frac{(f_{\boldsymbol{\theta}} - 1) \phi_{\boldsymbol{\theta}}}{\sum_{i \in \Gamma(\boldsymbol{\theta})} I_i} + \Lambda_{\boldsymbol{\theta}} (f_{\boldsymbol{\theta}} - 1) \left( \frac{S_{\boldsymbol{\theta}}}{\sum_{i \in \Gamma(\boldsymbol{\theta})} I_i} \right) = 0,$$

where  $w_i^{\boldsymbol{\theta}} = \frac{I_i}{\sum_{i \in \Gamma(\boldsymbol{\theta})} I_i}$ . The Kuhn-Tucker conditions imply that, for all  $\boldsymbol{\theta}$ :

$$\begin{aligned} (f_{\boldsymbol{\theta}} - 1) \phi_{\boldsymbol{\theta}} &= 0, \\ \Lambda_{\boldsymbol{\theta}} (f_{\boldsymbol{\theta}} - 1) \left( \frac{S_{\boldsymbol{\theta}}}{\sum_{i \in \Gamma(\boldsymbol{\theta})} I_i} \right) &= 0. \end{aligned}$$

Thus, we may write:

$$\sum_{i \in \Gamma(\boldsymbol{\theta})} \omega_{\Theta}(\boldsymbol{\theta}) (f_{\boldsymbol{\theta}} - 1) w_i^{\boldsymbol{\theta}} \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \pi^{\boldsymbol{\theta}} \right) - \sigma_{\boldsymbol{\theta}} (f_{\boldsymbol{\theta}} - 1) = 0. \quad (42)$$

Subtracting the left-hand side of  $[I_i]$  from  $[B_i]$  yields:

$$R_i = \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} \omega_{\Theta}(\boldsymbol{\theta}) (1 - f_{\boldsymbol{\theta}}) \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \mu \pi^{\boldsymbol{\theta}} \right) + \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}} f_{\boldsymbol{\theta}} \sigma_{\boldsymbol{\theta}} - \mu \delta_B \leq 0 \quad (43)$$

Note that  $\sum_{\boldsymbol{\theta} \in \Theta^i} \equiv \sum_{\mathbf{s} \in \Psi} \sum_{\boldsymbol{\theta} \in \Upsilon_i^{\mathbf{s}^1}}$ . Summing (42) over  $\boldsymbol{\theta} \in \Theta^i$  and adding it to (43) yields:

$$R_i = \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\Theta}(\boldsymbol{\theta}) (1 - f_{\boldsymbol{\theta}}) \left( \frac{\partial U_i^{\boldsymbol{\theta}}}{\partial W} - \sum_{j \in \Gamma(\boldsymbol{\theta})} w_j^{\boldsymbol{\theta}} \frac{\partial U_j^{\boldsymbol{\theta}}}{\partial W} \right) + \sum_{\boldsymbol{\theta} \in \Theta^i} \sigma_{\boldsymbol{\theta}} - \mu \delta_B \leq 0 \quad (44)$$

Subtracting the left-hand side of [A] yields:

$$R_i = \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta)(1 - f_{\theta}) \left( \frac{\partial U_i^{\theta}}{\partial W} - \sum_{j \in \Gamma(\theta)} w_j^{\theta} \frac{\partial U_j^{\theta}}{\partial W} \right) - \sum_{\theta \notin \Theta^i} \sigma_{\theta} + \mu(\delta_A - \delta_B) \leq 0 \quad (45)$$

This is the final expression for the bond return.

### C.3 Characterization of Solution without Frictional Costs

This section presents four lemmas in support of the claims in Section C.1. Specifically, the intention is to show that, in the absence of frictional costs, 1) full insurance is optimal, and 2) given full insurance, the division of risk transfer between reinsurance policies and catastrophe bonds is irrelevant. The approach is as follows.

The first lemma facilitates later work. It shows that, for each consumer, the sum of promised indemnification under insurance policies and catastrophe bond principal in any solution will be at least equal to the loss, allowing us to rule out solutions where some consumers have less than full coverage.

The second lemma shows that, if the optimal solution features exactly full coverage for each consumer (in the sense of the sum of promised indemnification under insurance policies and catastrophe bond principal being exactly equal to the loss), there will be no default: In other words, full coverage means full insurance.

The third lemma shows the indeterminacy of risk transfer: If consumers have exactly full coverage (and hence, by the second lemma, full insurance), it does not matter how that coverage is delivered—a continuum of solutions with exactly full coverage (but different splits of coverage between cat bonds and insurance) exist.

The fourth lemma shows that, in any case where a solution exists where a consumer has greater than full coverage (i.e., the sum of promised indemnity and catastrophe bond principal is *greater* than the loss), an equivalent solution can be constructed with exactly full coverage. This completes the proof of the “full insurance is always optimal.” By the first lemma, we know that only solutions with exactly full coverage and greater than full coverage need be considered. By the fourth lemma, we know that any solution with greater than fully coverage can be replicated by a solution with exactly full coverage. By the second lemma, we know that a solution with full coverage also features no default (i.e., full insurance).

Likewise, the fourth lemma completes the proof of the second claim—that the division of risk transfer between insurance policies and catastrophe bonds is indeterminate in the absence of frictional costs. By the reasoning in the preceding paragraph, we know that solutions with exactly full coverage and full insurance are always optimal in the absence of frictional costs. By the third lemma, we know that the division of risk transfer between insurance policies and catastrophe bonds is indeterminate in solutions with exactly full coverage and full insurance, in the absence of frictional costs.

**Lemma.** *Consider the maximization of (30) subject to the constraints below it, and assume at least one solution exists. Then there exists a solution  $\hat{A}, \{\hat{B}_i\}, \{\hat{c}_i\}, \{\hat{I}_i\}, \{\hat{f}_{\theta}\}, \{\hat{W}_{is}\}$  with  $\hat{I}_i + \hat{B}_i \geq L_i$ , for all  $i$ .*

**Proof:**

Suppose not. Then there is a solution with  $\hat{I}_k + \hat{B}_k < L_k$  for some  $k \in \{1, \dots, N\}$ . Consider an alternative set of choices:

$$\tilde{A} = \hat{A}.$$

$$\tilde{I}_i = \hat{I}_i, \text{ for all } i.$$

$$\tilde{f}_\theta = \hat{f}_\theta, \text{ for all } \theta.$$

$$\tilde{B}_i = \hat{B}_i, \text{ for all } i \neq k; \quad \tilde{B}_k = \hat{B}_k + \left( L_k - (\hat{I}_k + \hat{B}_k) \right).$$

$$\tilde{c}_i = \hat{c}_i, \text{ for all } i \neq k; \quad \tilde{c}_k = \hat{c}_k + \sum_{\theta \in \Theta^k} \omega_\Theta(\theta) \pi^\theta \left( \tilde{B}_k - \hat{B}_k \right)$$

$$\tilde{W}_{is} = \hat{W}_{is}, \forall i \neq k, \forall s; \quad \tilde{W}_{ks} = \hat{W}_{ks} - \sum_{\theta \in \{\theta: \theta(N+1)=s, \theta \in \Theta^k\}} \omega_\Theta(\theta) \left( \tilde{B}_k - \hat{B}_k \right), \forall s$$

We first verify that these new choices satisfy all constraints. First, note that

$$\begin{aligned} & \sum_{\theta \in \Theta} \left( \omega_\Theta(\theta) \pi^\theta \tilde{f}_\theta \sum_{i \in \Gamma(\theta)} \tilde{I}_i \right) + \sum_i \left( \sum_{\theta \in \Theta^i} \omega_\Theta(\theta) \pi^\theta \right) \tilde{B}_i = \\ & \sum_{\theta \in \Theta} \left( \omega_\Theta(\theta) \pi^\theta \hat{f}_\theta \sum_{i \in \Gamma(\theta)} \hat{I}_i \right) + \sum_i \left( \sum_{\theta \in \Theta^i} \omega_\Theta(\theta) \pi^\theta \right) \hat{B}_i + \sum_{\theta \in \Theta^k} \omega_\Theta(\theta) \pi^\theta \left( \tilde{B}_k - \hat{B}_k \right) \leq \\ & \sum \hat{c}_i + \sum_{\theta \in \Theta^k} \omega_\Theta(\theta) \pi^\theta \left( \tilde{B}_k - \hat{B}_k \right) = \sum \tilde{c}_i. \end{aligned}$$

So  $[\mu]$  is satisfied.

Next, since  $\tilde{A} = \hat{A}$ ,  $\tilde{I}_i = \hat{I}_i$  (for all  $i$ ), and  $\tilde{f}_\theta = \hat{f}_\theta$  (for all  $\theta$ ), the new choices satisfy  $[\lambda_\theta]$ ,  $[\Lambda_\theta]$ , and  $[\phi_\theta]$ .

Next, note that  $[\varphi_i]$  is obviously satisfied for  $i \neq k$ . For  $i = k$ , note that:

$$\begin{aligned} \sum_{s \in \Psi} \pi_s \tilde{W}_{ks} &= \sum_{s \in \Psi} \pi_s \left( \hat{W}_{ks} - \sum_{\theta \in \Upsilon_k^{s1}} \omega_\Theta(\theta) \left( \tilde{B}_k - \hat{B}_k \right) \right) \\ &\leq W - \hat{c}_k - \sum_{s \in \Psi} \pi_s \sum_{\theta \in \Upsilon_k^{s1}} \omega_\Theta(\theta) \left( \tilde{B}_k - \hat{B}_k \right) = W - \tilde{c}_k, \end{aligned}$$

so  $[\varphi_k]$  is satisfied.

The non-negativity constraints are obviously satisfied in all cases.

We next verify that the proposed alternative set of choices improves the objective function. Since  $\tilde{I}_i = \hat{I}_i$  (for all  $i$ ),  $\tilde{B}_i = \hat{B}_i$  (for all  $i \neq k$ ),  $\tilde{c}_i = \hat{c}_i$  (for all  $i \neq k$ ),  $\tilde{W}_{is} = \hat{W}_{is}$  (for all  $i \neq k$ ), and  $\tilde{f}_\theta = \hat{f}_\theta$  (for all  $\theta$ ), it is obvious that the alternative choices yield identical levels of expected utility for all consumers other than consumer  $k$ .

Recall the definitions  $\Upsilon_k^{s0} \equiv \{\theta : \theta \notin \Theta^k, \theta(N+1) = s\}$  and  $\Upsilon_k^{s1} \equiv \{\theta : \theta \in \Theta^k, \theta(N+1) = s\}$ , noting further that

$$\Upsilon^s \equiv \Upsilon_k^{s0} \cup \Upsilon_k^{s1} = \{\theta : \theta(N+1) = s\}.$$

For consumer  $k$ ,

$$\begin{aligned}
E\tilde{U}_k &= \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k \right) + \sum_{s \in \Psi} \sum_{\theta \notin \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} \right) \\
E\hat{U}_k &= \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) + \sum_{s \in \Psi} \sum_{\theta \notin \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} \right)
\end{aligned}$$

The difference  $E\tilde{U}_k - E\hat{U}_k$  reflects the increase in utility when moving from the “solution” to the utility level obtained with the alternative set of choices. It can be written as:

$$\begin{aligned}
&\sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) \right] + \\
&\sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_{ks} \right) - U_k \left( \hat{W}_{ks} \right) \right]
\end{aligned}$$

If this difference is weakly positive, then the alternative set of choices constitutes an additional solution, and we have our desired contradiction.

If security markets are complete with respect to consumer  $k$ 's loss exposure (i.e., if, for every  $s$ , either  $\Upsilon_k^{s0} = \emptyset$  or  $\Upsilon_k^{s1} = \emptyset$ ), then it is trivial to show that this difference is zero. If security markets are incomplete with respect to consumer  $k$ 's loss exposure, then there will be some nonempty subset  $Z \subset \Psi$  such that:

$$Z = \{s : \Upsilon_k^{s0} \neq \emptyset, \Upsilon_k^{s1} \neq \emptyset\}.$$

For every  $s \in Z$ ,

$$\begin{aligned}
&\sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) \right] + \\
&\sum_{\theta \in \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_{ks} \right) - U_k \left( \hat{W}_{ks} \right) \right]
\end{aligned}$$

may be rewritten as:

$$\begin{aligned}
&\sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k \right) + \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} \right) - \\
&\sum_{\theta \in \Upsilon_k^{s1}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) + \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} \right)
\end{aligned}$$

and further as:

$$\begin{aligned}
& \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) (\tilde{B}_k - \hat{B}_k) - L_k + \tilde{f}_{\boldsymbol{\theta}} \tilde{I}_k + (\tilde{B}_k - \hat{B}_k) + \tilde{B}_k \right) \\
& + \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) (\tilde{B}_k - \hat{B}_k) \right) - \\
& \left( \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k (\hat{W}_{ks} - L_k + \hat{f}_{\boldsymbol{\theta}} \hat{I}_k + \hat{B}_k) + \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k (\hat{W}_{ks}) \right),
\end{aligned}$$

which is greater than zero, since  $U_k$  is concave and

$$\hat{f}_{\boldsymbol{\theta}} \hat{I}_k + \hat{B}_k < \tilde{f}_{\boldsymbol{\theta}} \tilde{I}_k + (\tilde{B}_k - \hat{B}_k) + \tilde{B}_k \leq L_k, \forall \boldsymbol{\theta}.$$

QED.

**Lemma.** Let  $\hat{A}, \{\hat{B}_i\}, \{\hat{c}_i\}, \{\hat{I}_i\}, \{\hat{f}_{\boldsymbol{\theta}}\}, \{\hat{W}_{is}\}$  be choices that solve (30) subject to the constraints below it, and suppose  $\hat{I}_i + \hat{B}_i = L$ , for all  $i$ . Then there exists a solution  $\tilde{A}, \{\tilde{B}_i\}, \{\tilde{c}_i\}, \{\tilde{I}_i\}, \{\tilde{f}_{\boldsymbol{\theta}}\}, \{\tilde{W}_{is}\}$  with 1)  $\tilde{I}_i + \tilde{B}_i = L_i$ , for all  $i$ , and 2)  $\tilde{f}_{\boldsymbol{\theta}} = 1$ , for all  $\boldsymbol{\theta}$  where  $\Gamma(\boldsymbol{\theta})$  is non-empty and  $\hat{I}_i > 0$  for some  $i \in \Gamma(\boldsymbol{\theta})$ .

**Proof:**

Suppose not. Then the set  $\Delta = \{\boldsymbol{\theta} : \hat{f}_{\boldsymbol{\theta}} < 1\}$  is nonempty and  $\hat{I}_i > 0$  for some  $i \in \Gamma(\boldsymbol{\theta})$  for some  $\boldsymbol{\theta} \in \Delta$ .

Let  $\Phi \equiv \{i : \hat{I}_i > 0, i \in \Gamma(\boldsymbol{\theta}) \text{ for some } \boldsymbol{\theta} \in \Delta\}$ . Consider an alternative set of choices as follows:

$$\tilde{f}_{\boldsymbol{\theta}} = 1, \forall \boldsymbol{\theta}.$$

$$\tilde{I}_i = \hat{I}_i, \text{ for all } i.$$

$$\tilde{B}_i = \hat{B}_i, \text{ for all } i$$

$$\tilde{A} = \max \left[ \hat{A}, \max_{\boldsymbol{\theta} \in \Delta} \sum_{i \in \Gamma(\boldsymbol{\theta})} \tilde{I}_i \right]$$

$$\tilde{c}_i = \hat{c}_i + \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i, \forall i.$$

$$\tilde{W}_{is} = \hat{W}_{is} - \sum_{\boldsymbol{\theta} \in \{\boldsymbol{\theta} : \boldsymbol{\theta} \in \Theta^i, \boldsymbol{\theta}(N+1)=s\}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i, \forall s, \forall i.$$

We first verify that the constraints are satisfied.

Starting with  $[\mu]$ , note that

$$\begin{aligned}
\sum \tilde{c}_i &= \sum_i \left( \hat{c}_i + \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i \right) \\
&= \sum_{\boldsymbol{\theta} \in \Theta} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \hat{f}_{\boldsymbol{\theta}} \sum_{i \in \Gamma(\boldsymbol{\theta})} \hat{I}_i + \sum_i \left( \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \right) \hat{B}_i + \sum_i \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i \\
&= \sum_{\boldsymbol{\theta} \in \Theta} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \tilde{f}_{\boldsymbol{\theta}} \sum_{i \in \Gamma(\boldsymbol{\theta})} \tilde{I}_i + \sum_i \left( \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \right) \tilde{B}_i.
\end{aligned}$$

By construction of  $\tilde{A}$ , it is clear that  $[\lambda_{\boldsymbol{\theta}}]$  is satisfied. Likewise, since  $\tilde{f}_{\boldsymbol{\theta}} = 1, \forall \boldsymbol{\theta}$ ,  $[\phi_{\boldsymbol{\theta}}]$  and  $[\Lambda_{\boldsymbol{\theta}}]$  are satisfied.

For the individual wealth constraint  $[\varphi_i]$ ,

$$\begin{aligned}
\sum_s \pi_s \tilde{W}_{is} &= \sum_s \pi_s \hat{W}_{is} - \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i \\
&\leq W - \hat{c}_i - \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\Theta}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} (\tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}}) \tilde{I}_i \\
&= W - \tilde{c}_i.
\end{aligned}$$

The non-negativity constraints are obviously satisfied. We now must verify that the alternative choices improve the objective function.

Evidently, utility is unchanged for  $i \notin \Phi$ , so we are left to verify that utility improves for  $i \in \Phi$ . We thus consider some consumer  $k$  (with  $k \in \Phi$ ). To verify that utility improves with the new choices, we focus on the utility levels under the alternative and original choices, as in:

$$\begin{aligned}
E\tilde{U}_k &= \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\Theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\boldsymbol{\theta}} \tilde{I}_k + \tilde{B}_k \right) + \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\Theta}(\boldsymbol{\theta}) U_k \left( \tilde{W}_{ks} \right) \\
E\hat{U}_k &= \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\Theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_k^{\boldsymbol{\theta}} - L_k + \hat{f}_{\boldsymbol{\theta}} \hat{I}_k + \hat{B}_k \right) + \sum_s \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\Theta}(\boldsymbol{\theta}) U_k \left( \hat{W}_k^{\boldsymbol{\theta}} \right)
\end{aligned}$$

and examine the difference  $E\tilde{U}_k - E\hat{U}_k$ .

If security markets are complete with respect to consumer  $k$ 's loss exposure (i.e., if, for every  $s$ , either  $\Upsilon_k^{s0} = \emptyset$  or  $\Upsilon_k^{s1} = \emptyset$ ), then it can be shown that this difference is zero—since security markets can replicate the payoffs from insurance policies for consumer  $k$ . Similarly, when security markets are incomplete, but the sets  $Z \equiv \{s : \Upsilon_k^{s0} \neq \emptyset, \Upsilon_k^{s1} \neq \emptyset\}$  and  $X = \{\boldsymbol{\theta}(N+1) : \boldsymbol{\theta} \in \Delta\}$  do not intersect, the difference will be zero for the same reason.

On the other hand, if  $Z \cap X \neq \emptyset$ , then consumer  $k$  cannot replicate insurance policy payoffs with securities. For every  $s$ ,

$$\begin{aligned} & \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\boldsymbol{\theta}} \tilde{I}_k + \tilde{B}_k \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\boldsymbol{\theta}} \hat{I}_k + \hat{B}_k \right) \right] + \\ & \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \left[ U_k \left( \tilde{W}_k^{\boldsymbol{\theta}} \right) - U_k \left( \hat{W}_k^{\boldsymbol{\theta}} \right) \right] \end{aligned}$$

may be rewritten as:

$$\begin{aligned} & \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \left( 1 - \hat{f}_{\boldsymbol{\theta}} \right) \hat{I}_k \right) + \\ & \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \left( 1 - \hat{f}_{\boldsymbol{\theta}} \right) \hat{I}_k \right) - \\ & \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} - \left( 1 - \hat{f}_{\boldsymbol{\theta}} \right) \hat{I}_k \right) + \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s0}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) U_k \left( \hat{W}_{ks} \right) \end{aligned}$$

which is greater than or equal to zero, since  $U_k$  is concave, with strict inequality when  $s \in Z \cap X$ .

QED.

**Lemma.** Let  $\hat{A}, \{\hat{B}_i\}, \{\hat{c}_i\}, \{\hat{I}_i\}, \{\hat{f}_{\boldsymbol{\theta}}\}, \{\hat{W}_{is}\}$  be choices that solve (30) subject to the constraints below it, and  $\hat{I}_i + \hat{B}_i = L$ , for all  $i$ . Then, an equivalent solution yielding 1) the same value for the overall objective function and 2) the same values for the utility levels of each of the individual consumers, can be constructed as  $\hat{A}, \{\tilde{B}_i\}, \{\tilde{c}_i\}, \{\tilde{I}_i\}, \{\tilde{f}_{\boldsymbol{\theta}}\}, \{\tilde{W}_{is}\}$ , where  $\{\tilde{B}_i\}$  and  $\{\tilde{I}_i\}$  are any alternative sets of feasible choices satisfying the nonnegativity constraints and  $\tilde{I}_i + \tilde{B}_i = L_i$ , for all  $i$ , and  $\hat{A} = \max_{\boldsymbol{\theta}} \sum_{i \in \Gamma(\boldsymbol{\theta})} \tilde{I}_i$ .

**Proof:**

The preceding lemma shows that for any solution with  $\hat{I}_i + \hat{B}_i = L_i$ , for all  $i$ , an equivalent solution can be constructed as:

$$\tilde{f}_{\boldsymbol{\theta}} = 1, \forall \boldsymbol{\theta}.$$

$$\tilde{I}_i = \hat{I}_i, \text{ for all } i.$$

$$\tilde{B}_i = \hat{B}_i, \text{ for all } i$$

$$\tilde{A} = \max \left[ \hat{A}, \max_{\boldsymbol{\theta} \in \Delta} \sum_{i \in \Gamma(\boldsymbol{\theta})} \tilde{I}_i \right]$$

$$\tilde{c}_i = \hat{c}_i + \sum_{\boldsymbol{\theta} \in \Theta^i} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \pi^{\boldsymbol{\theta}} \left( \tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}} \right) \tilde{I}_i, \forall i.$$

$$\tilde{W}_{is} = \hat{W}_{is} - \sum_{\boldsymbol{\theta} \in \Upsilon_k^{s1}} \omega_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \left( \tilde{f}_{\boldsymbol{\theta}} - \hat{f}_{\boldsymbol{\theta}} \right) \tilde{I}_i, \forall s, \forall i.$$

Then, given any sets  $\{\tilde{B}_i\}$  and  $\{\tilde{I}_i\}$  that satisfy  $\tilde{I}_i + \tilde{B}_i = L_i$ ,  $\tilde{I}_i \geq 0$ , and  $\tilde{B}_i \geq 0$ , for all  $i$ , we construct the remaining choice variables as:

$$\tilde{f}_{\boldsymbol{\theta}} = 1, \forall \boldsymbol{\theta}.$$

$$\tilde{A} = \max_{\boldsymbol{\theta}} \sum_{i \in \Gamma(\boldsymbol{\theta})} \tilde{I}_i$$

$$\begin{aligned}\tilde{c}_i &= \check{c}_i, \quad \forall i. \\ \tilde{W}_{is} &= \check{W}_{is}, \quad \forall s, \forall i.\end{aligned}$$

We now must show that the new choices satisfy all constraints and (weakly) improve the objective function. Starting with constraint  $[\mu]$ , in the absence of frictional costs, we have:

$$\sum \tilde{c}_i = \sum \check{c}_i \geq \sum_{\theta \in \Theta} \left( \omega_{\theta}(\boldsymbol{\theta}) \pi^{\theta} f_{\theta} \sum_{i \in \Gamma(\theta)} \check{I}_i \right) + \sum_i \left( \sum_{\theta \in \Theta^i} \omega_{\theta}(\boldsymbol{\theta}) \pi^{\theta} \right) \check{B}_i$$

Since  $f_{\theta} = 1$ , we may rewrite as:

$$\sum \tilde{c}_i = \sum \check{c}_i \geq \sum_i (\omega_{\theta}(\boldsymbol{\theta}) \pi^{\theta}) (\check{B}_i + \check{I}_i)$$

But, since

$$\check{B}_i + \check{I}_i = \tilde{I}_i + \tilde{B}_i = L,$$

$$\sum \tilde{c}_i \geq \sum_i \left( \sum_{\theta \in \Theta^i} \omega_{\theta}(\boldsymbol{\theta}) \pi^{\theta} \right) (\tilde{B}_i + \tilde{I}_i),$$

so the constraint is satisfied.

For constraint  $[\lambda_{\theta}]$ , note that:

$$\tilde{A} = \max_{\theta} \sum_{i \in \Gamma(\theta)} \tilde{I}_i,$$

which is obviously greater than or equal to  $f_{\theta} \sum_{i \in \Gamma(\theta)} \tilde{I}_i$  for all  $\theta$ , so the constraint is satisfied.

Constraints  $[\phi_{\theta}]$ ,  $[\Lambda_{\theta}]$  and  $[\varphi_i]$  are obviously satisfied, since  $\tilde{f}_{\theta} = 1$ ,  $\tilde{c}_i = \check{c}_i$ , and  $\tilde{W}_{is} = \check{W}_{is}$ .

Since 1) state contingent wealth allocations are unchanged under the new choices ( $\tilde{W}_{is} = \check{W}_{is}$ , for all  $s$  and for all  $i$ ), and 2) the total of insurance policy and catastrophe bond recoveries are unchanged for every consumer in every state of the world where that consumer experiences a loss:

$$f_{\theta} \tilde{I}_i + \tilde{B}_i = \tilde{I}_i + \tilde{B}_i = \check{I}_i + \check{B}_i = f_{\theta} \check{I}_i + \check{B}_i, \quad \forall i, \theta \in \Theta^i,$$

it is evident that utility is unchanged for every consumer. Thus, the objective function is unchanged and the alternative choices proposed must constitute an additional solution.

QED.

**Lemma.** *Let  $\hat{A}$ ,  $\{\hat{B}_i\}$ ,  $\{\hat{c}_i\}$ ,  $\{\hat{I}_i\}$ ,  $\{\hat{f}_{\theta}\}$ ,  $\{\hat{W}_{is}\}$  be choices that solve (30) subject to the constraints below it, and suppose  $\hat{I}_i + \hat{B}_i > L$ , for at least one  $i$ . Then there exists a “full coverage, full insurance” solution yielding the same value for the overall objective function that can be constructed with  $\{\tilde{B}_i\}$  and  $\{\tilde{I}_i\}$  as alternative sets of feasible choices satisfying the nonnegativity constraints and  $\tilde{I}_i + \tilde{B}_i = L_i$ , for all  $i$ ,  $\tilde{f}_{\theta} = 1 \forall \theta$ , and  $\tilde{A} = \max_{\theta} \sum_{i \in \Gamma(\theta)} \tilde{I}_i$ .*

**Proof:**

From the preceding lemma, we know that all solutions with  $\tilde{I}_i + \tilde{B}_i = L_i$ , for all  $i$ , and  $\tilde{f}_\theta = 1, \forall \theta$ , are equivalent. Given sets  $\{\tilde{B}_i\}$  and  $\{\tilde{I}_i\}$  satisfying these conditions, construct the remaining choice variables as:

$$\tilde{A} = \max_{\theta} \sum_{i \in \Gamma(\theta)} \tilde{I}_i$$

$$\tilde{f}_\theta = 1, \forall \theta$$

$$\tilde{c}_i = \hat{c}_i + \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right]$$

$$\tilde{W}_{is} = \hat{W}_{is} - \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right], \forall s, \forall i$$

We start by verifying that these alternative choices satisfy the constraints. First, rewrite the original constraint as:

$$\sum \hat{c}_i \geq \sum_i \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left( \hat{f}_\theta \hat{I}_i + \hat{B}_i \right).$$

Furthermore,

$$\begin{aligned} \sum \tilde{c}_i &= \sum \hat{c}_i + \sum_i \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right] \\ &\geq \sum_i \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left( \hat{f}_\theta \hat{I}_i + \hat{B}_i \right) + \sum_i \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right] \\ &\geq \sum_i \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \left( \tilde{f}_\theta \tilde{I}_i + \tilde{B}_i \right) \\ &= \sum_{\theta \in \Theta} \left( \omega_{\Theta}(\theta) \pi^\theta \tilde{f}_\theta \sum_{i \in \Gamma(\theta)} \tilde{I}_i \right) + \sum_i \left( \sum_{\theta \in \Theta^i} \omega_{\Theta}(\theta) \pi^\theta \right) \tilde{B}_i \end{aligned}$$

so  $[\mu]$  is satisfied.

Next, since  $\tilde{A} = \max_{\theta} \sum_{i \in \Gamma(\theta)} \tilde{I}_i$  and  $\tilde{f}_\theta = 1$  for all  $\theta$ ,  $[\lambda_\theta]$ ,  $[\Lambda_\theta]$  and  $[\phi_\theta]$  are obviously satisfied.

For  $[\varphi_i]$ , note that, for each  $i$ ,

$$\begin{aligned} \sum_{s \in \Psi} \pi_s \tilde{W}_{is} &= \sum_{s \in \Psi} \pi_s \hat{W}_{is} - \sum_{s \in \Psi} \pi_s \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right] \\ &\leq W - \hat{c}_i - \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \pi^\theta \left[ \left( \tilde{f}_\theta \tilde{I}_i - \hat{f}_\theta \hat{I}_i \right) + \left( \tilde{B}_i - \hat{B}_i \right) \right] \\ &= W - \tilde{c}_i, \end{aligned}$$

so the constraints are evidently satisfied.

By definition, the non-negativity constraints are satisfied.

It remains to show that the objective function (weakly) improves under the alternative choices. For any consumer  $k$ , the difference between utility achieved under the alternative choices and the original choices can be expressed as:

$$\begin{aligned} & \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ U_k \left( \tilde{W}_{ks} - L_k + \tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) \right] + \\ & \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\Theta}(\theta) \left[ U_k \left( \tilde{W}_{ks} \right) - U_k \left( \hat{W}_{ks} \right) \right]. \end{aligned}$$

Simplifying (recall that  $\tilde{f}_{\theta} \tilde{I}_k + \tilde{B}_k$  always equals  $L$ ) yields:

$$\begin{aligned} & \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ U_k \left( \tilde{W}_{ks} \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) \right] + \\ & \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\Theta}(\theta) \left[ U_k \left( \tilde{W}_{ks} \right) - U_k \left( \hat{W}_{ks} \right) \right], \end{aligned}$$

or:

$$\begin{aligned} & \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ U_k \left( \hat{W}_{ks} - \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ L_k - (\hat{f}_{\theta} \hat{I}_k + \hat{B}_k) \right] \right) - U_k \left( \hat{W}_{ks} - L_k + \hat{f}_{\theta} \hat{I}_k + \hat{B}_k \right) \right] \\ & + \sum_{s \in \Psi} \sum_{\theta \in \Upsilon_k^{s0}} \omega_{\Theta}(\theta) \left[ U_k \left( \hat{W}_{ks} - \sum_{\theta \in \Upsilon_k^{s1}} \omega_{\Theta}(\theta) \left[ L_k - (\hat{f}_{\theta} \hat{I}_k + \hat{B}_k) \right] \right) - U_k \left( \hat{W}_{ks} \right) \right] \end{aligned}$$

Since  $U_k$  is concave, it follows that this difference is weakly greater than zero.

QED.

## D Data

### D.1 Data on Exposure

In a 2006 study (cited in the body of the paper), AIR Worldwide Corporation calculated that the insurance industry's total US property exposure was approximately equal to \$44 trillion. The same report concluded that exposure in coastal counties from Texas to Maine (subject to Hurricane risk) neared \$7 trillion. Note that these figures refer only to property exposures; including liability would add significant exposures to the total.

### D.2 Data on Collateralization

In the June 7, 2007 release of the *Flow of Funds Accounts of the United States*, the Federal Reserve reported total assets and capital of the domestic property-casualty insurance industry to be about \$1.25 trillion and \$450 billion, respectively, as of the end of 2005 (Table L.116). Some of the risks underwritten by domestic companies are, of course, transferred to offshore reinsurance companies—necessitating an estimate of risk-bearing assets held in the

international reinsurance markets. The International Association of Insurance Supervisors (IAIS) provided such an estimate in its *Global Reinsurance Market Report 2006*. The IAIS estimate of about \$800 billion for invested assets at market value at the end of 2005 (Table 5.1, Page 63) counts both U.S. and non-U.S. reinsurers. The same report puts total *capital* of non-U.S. reinsurers is about \$200 billion. Thus, it seems safe to conclude that \$2 trillion and \$650 billion are upper bounds on the risk-bearing assets and capital, respectively, held by the worldwide insurance and reinsurance industry in support of U.S. risks.

### D.3 Data on Frictional Costs of Catastrophe Bonds

A convenient source for catastrophe bond transaction data is the annual review of the market produced by Lane Financial, L.L.C. We examined the transactional data reported in the reviews published in 2000 to 2007, covering new issues from March, 1999 to March, 2007. We obtained usable data (defined as any transaction for which issue size, adjusted spread premium, and expected excess return were available) for 180 non-life transactions, totalling over \$13 billion in issuance. The key cost variable in this data is the *expected excess return*, which is calculated as the bond yield's annualized spread over LIBOR, minus the annualized expected loss on the bond.

For the entire sample, the average expected excess return (weighted by issue size) is 535 basis points. To this must be added an estimate of issuance costs—such as up-front underwriting fees, rating agency fees, legal fees, and catastrophe modelling fees. According to the GAO, these can amount to 2% of the issue.<sup>23</sup> Given that the weighted average exposure term for the sample was about 34 months, this means that the annualized issuance costs would add about 70 basis points per year ( $[200/34] * 12 \approx 70$ ) to the annual frictional cost. Thus, the total frictional cost estimate is  $535+70=605$  basis points. There are two important caveats to this estimate.

First, the estimate does not include any estimate of the costs stemming from basis risk borne by the issuer. Since most catastrophe bond deal feature non-indemnity triggers, these costs could be significant. As will be noted below, the insurance industry as a whole incurs significant loss adjustment expenses due, in large part, to the indemnity triggers featured in the contracts.

Second, the evidence strongly suggests that the expected excess return on catastrophe bonds rises with the risk of loss. This has been noted previously by Lane [18], who, after observing the connection between expected excess return and expected loss, suggested a pricing model based on a decomposition of expected loss. The connection is strongly evident in the data. As an illustration, the sample average expected loss (weighted by issue size) is 134 basis points—corresponding to an expected excess return of 535 basis points. If we restrict our attention, however, to the 45 deals with expected loss of 200 basis points or more, the corresponding subsample average expected excess return rises to 1093 basis points. If we further restrict analysis to the 25 deals with expected loss of 400 basis points or more, the figure rises to 1596 basis points.

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<sup>23</sup>United States General Accounting Office (September, 2003), “Catastrophe Insurance Risks: Status of Efforts to Securitize Natural Catastrophe and Terrorism Risk,” *Report to Congressional Requesters* (GAO-03-1033), Page 21.

## D.4 Data on Frictional Costs of Reinsurance

Relative to the previous exercise, the measurement of the frictional costs associated with reinsurance assets is much more complicated and open to debate. Defining the relevant costs and assets is challenging.

On the cost side, we decompose the frictional cost of reinsurance into the underwriting expenses of the reinsurer, which can be measured directly, and the expected excess return on the equity capital of the reinsurer, which must be imputed. Calculating the latter is the less straightforward of the two tasks.

Cummins and Phillips [7] use several methods to estimate the cost of equity capital for property-liability insurers; after deducting the risk-free rate, their results imply a range for the expected excess return—from a low of about 700 basis points to about 1400 basis points. It should be noted that Cummins and Phillips use a long-run equity premium of 8.44% as an input, which is justifiable but a fairly aggressive choice (see Welch [26]). An examination of the recent profit targets of some leading insurers provides support for the lower end of the range—perhaps 750 to 1000 basis points.<sup>24</sup> For our analysis, we use a range of 700 to 1300 basis points.

To estimate reinsurer expenses, we start with A.M. Best’s *Aggregates and Averages*, which contains annual data for a subset of companies designated as “Professional Reinsurers” covering the period 2000 to 2005; the data is available in the Quantitative Analysis Report sections for the 2006 and 2005 publication years. To estimate total expenses for a given year, we add Best’s figures for underwriting expenses incurred and income taxes to the imputed cost of equity—which we calculate as the sum of 1) the product of an assumed expected excess return (see above) and the total industry surplus available at the start of the year and 2) the product of 5% (an estimate of underwriting fees) and Best’s figure for the industry total for “Contributed Capital.” This total for frictional expenses forms the numerator of a frictional cost ratio for reinsurance assets.

The more conceptually demanding task concerns determining what to put in the denominator—i.e., the appropriate definition of “assets.” First, the total assets held by reinsurers typically support risks beyond those associated with exposures in a given year; in particular, the reinsurer holds loss reserves for losses incurred in previous years. But this presents a pitfall—a reinsurer serving a “long-tailed” line may, *ceteris paribus*, be holding substantially more assets than a “short-tailed” counterpart despite having the same annual flow of risk. The difference is that the losses experienced by the former take longer to settle. To sidestep this issue, we define available assets according to the surplus available at the start of the year plus the flow of funds generated during the year—the sum of premiums written, net investment income, realized and unrealized capital gains, other surplus gain or loss, and contributed capital; minus dividends, underwriting expenses, and taxes. A second issue concerns the treatment of reinsurance and retrocession coverage purchased by the reinsurers, which expands the asset base supporting their risks. To address this, we use gross premiums written rather than net premiums written (a similar result would be obtained if we used net premi-

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<sup>24</sup>The 2006 annual reports of the top five reinsurers were consulted, with three revealing an ROE target of some kind. Swiss Re disclosed an “over-the-cycle” target ROE of 13%; Munich Re disclosed an “over-the-cycle” target RORAC (return on risk-adjusted capital) of 15%; Hannover Re disclosed a minimum ROE target of the risk-free rate plus 750 basis points.

ums written and included “funds withheld under reinsurance contracts” in available assets); it should be noted that this significantly understates the likely recoveries under reinsurance contracts. This asset total forms the denominator of our frictional cost ratio.

A weighted average of our figures for the 2001-2005 period yields a range of estimates for the frictional cost ratio—from 1170 basis points (assuming a 700 basis point expected excess return on equity) to 1530 basis points (assuming a 1300 basis point expected excess return on equity). Given our inability to properly measure the extent of risk transfer through reinsurance/retrocession, these should be regarded as high-end estimates.

One could further argue that the figures are high because the underwriting expenses associated with excess-of-loss reinsurance (the type most directly comparable with catastrophe bonds) are lower than those for the industry as a whole. Using 2006 calendar year data from the Association of Bermuda Insurers and Reinsurers (using only those carriers without any direct writings), and 2000 through 2006 calendar year data from the Reinsurance Association of America, we come up with an estimate of 18% of net premiums written for underwriting expenses for catastrophe reinsurance—composed of the standard 10% commission rate on excess-of-loss reinsurance contracts in the Bermuda and an 8% rate for other underwriting expenses (based on the data cited above). We then use Bermuda Monetary Authority data for the 2005 calendar year to establish the relationship between gross premiums and net premiums for Class 4 Reinsurers, and assume the relations derived from Best’s data hold for the other quantities. This exercise yields slightly lower estimates—a range from 1050 to 1410 basis points, as the lower underwriting expenses are partially offset by less (observed) purchase of risk transfer.

Note that we have not included loss adjustment expenses in our calculation of frictional cost. This makes the reinsurance results comparable to the catastrophe bond figures, since catastrophe bonds are typically structured so as to avoid loss adjustment expenses by imposing basis risk on the issuers. Including loss adjustment expenses would add about 200 basis points to our figures.

We replicate the analysis for the U.S. property-casualty as a whole. The methodology follows that used above except that Direct Premiums Written are used instead of Gross Premiums Written, since the latter is distorted by transfers between affiliates within a group. The estimate range is 1870 basis points to 2170 basis points. Loss adjustment expenses would add another 700 basis points to those figures.

Finally, we examine direct evidence on the pricing of property reinsurance.<sup>25</sup> The pricing indications cover renewals on regional and national programs at 1/1/05, 1/1/06, and 1/1/07 for two levels of expected loss relative to exposure—2% and 8%. The regional program pricing for the 1/1/05 renewals was in the neighborhood of 500 basis points (for the 2% expected loss layer) and 1100 for the 8% layer—indicating a cost to the buyer of about 300 basis points in both cases. The post-Katrina average cost to the buyer over the 1/1/06 and 1/1/07 renewals for regional programs was about 400 basis points for the 2% layer and 550 basis points for the 8% layer. For national programs, the pre-Katrina cost was about 550 basis points for the 2% layer and 300 basis points for the 8% layer, but the post-Katrina averages for both layers rose to the 1200 to 1300 basis point range.

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<sup>25</sup>The following is presented in Guy Carpenter’s 2007 report *U.S. Reinsurance Renewals at January 1, 2007: Smooth Sailing Ahead?*

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