

# Life Insurance and the Life Cycle<sup>\*</sup>

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## Abstract

This paper explores life insurance holdings from a general equilibrium perspective. Drawing on the data explored in Chambers, Schlagenhauf, and Young (2003), we calibrate an overlapping generations lifecycle economy with incomplete asset markets to match facts regarding the uncertainty of income and demographics. We then estimate that life insurance holdings for the purpose of smoothing family consumption are so large that they constitute a puzzle from the perspective of standard economic theory. Furthermore, the welfare gains

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from a life insurance market are concentrated in the minds of households who use the real world market very little.

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*Failure of the head of a family to insure his or her life against a sudden loss of economic value through death or disability amounts to gambling with the greatest of life's values; and the gamble is a particularly mean one because, in the case of loss, the dependent family, and not the gambler must suffer the consequences.*

S. Huebner and K. Black, Jr., **Life Insurance**

## 1 Introduction

Simply put, the life insurance market is big. In terms of policy face values, the total size of this market in 1998 was 0.95 times annual GDP. Alternatively, in terms of expenditures LIMRA data reports \$212 billion in total premiums paid during 1998, and the BEA category "Expenses of Handling Life Insurance and Pension Plans" constitutes 1.4 percent of total consumption. The general perception, perhaps as a result of the marketing strategy of life insurance firms, is that households are holding an insufficient amount of life insurance - the quote from the textbook by Huebner and Black insinuates this, as do commercials that assert how frequently a widow falls to poverty income levels as the result of the untimely death of her spouse.<sup>1</sup> A recent study by Bernheim *et.al.* (2003) examines life insurance holdings in light of financial vulnerability and finds that more financially-vulnerable households seem to be underinsured.<sup>2</sup> This argument is difficult to assess because life insurance can be held for various reasons that are unrelated directly to financial insurance; those authors do not attempt to measure each motive separately.

There are two main classes of life insurance policies available. **Term** insurance requires premium payments over a specified period; if death occurs during the period, the household receives the face value payment, but receives nothing if a death does not occur. In contrast, **whole-life** insurance lasts until cancelled and accumulates cash value over the life of the policy; this cash value can be borrowed against to finance current expenditures. The net

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<sup>1</sup>For example, an advertising campaign that aired during the 2001 World Series claimed that the average widow under the age of 50 would use up her life insurance payment within nine months. Recently, Zick and Holden (2000) find evidence in the Survey of Income and Program Participation that widows face significant wealth declines upon the death of their spouse. See also Hurd and Wise (1989).

<sup>2</sup>See also Bernheim *et.al.* (2001,2002) and Gokhale and Kotlikoff (2002).

cash value is disbursed to the household upon termination of the policy. The face value is still paid in the event of a death. In this paper, we define **total** life insurance holdings as the sum of term life insurance plus the face value of whole life insurance minus the accumulated cash value of the whole-life policy. The accumulated cash value of a whole-life insurance policy is savings and should not be considered life insurance. In Chambers, Schlagenhauf, and Young (2003) we examine life insurance data from the Survey of Consumer Finances for 1995, 1998, and 2001 and document facts from a different number of perspectives. Some of the facts documented in that paper from the 1998 abstract of the SCF that are especially pertinent to issues examined in this paper are:

- The life insurance participation and holding levels increase monotonically in earnings, income and wealth.
- A comparison of the top quintile with the bottom quintile using earnings, income, or wealth indicates that average insurance holdings are 13.4 times larger, 11.9 times larger, and 6.33 times larger, respectively.
- As can be seen in Figure 1, the participation rate for life insurance peaks in the late 60s.
- Figure 2 indicates that the peak holding of life insurance occurs around age 50.
- The insurance participation rate for two worker families is 69.5 percent while the participation rate for one worker families is 67.7 percent.
- The average life insurance holdings of a one worker family exceeds the two worker family by \$6,338. Compared to a two worker family, a one worker family has essentially equal earnings, income, and wealth.
- The average single female widow has \$27,746 lower earnings, \$27,000 less income and \$161,610 less wealth compared to the average one worker family.

Life insurance potentially has several driving forces – among them bequests, tax issues, and most importantly to our paper, consumption-smoothing within the household. The purpose of this paper is to provide an estimate of the size of this last motive; in spirit our exercise is in the same vein as the

estimate of the size of precautionary savings in Aiyagari (1994) and Pijoan-Mas (2003) or the classic estimate of the contribution of technology shocks to business cycles found in Kydland and Prescott (1982).

In order to estimate the size of the consumption-smoothing motive we construct a dynamic overlapping generations model. The decision making unit is the household, which enters a period with a demographic state comprised of age, sex, marital status, and the number of children. Households face idiosyncratic uncertainty in their hourly wage as well as in their demographic state. To insulate themselves against these shocks, agents can accumulate interest-bearing assets and life insurance policies and supply labor to the market. A competitive life insurance industry determines the equilibrium price of the life insurance policies. Our model is calibrated to produce a wealth and earnings distribution consistent with the data and demographic shocks that match observed transition probabilities from the Center for Disease Control and the Census Bureau.

We require a model because our data does not contain critical pieces of information needed to investigate our question. First, the SCF does not collect data on the premium paid for a term policy, and it contains only limited information on whole-life policies that accumulate cash value. Specifically, the data does not identify whether the policy is individual or group, so that it is unclear how this question would be answered if the employer paid the premium. In addition, it does not identify who the policy covers, so that the pricing data would not be perfectly informative in any case. We focus on a general equilibrium model, rather than a partial equilibrium one, because the endogenous relationship between the return on savings and the amount of self-insurance through financial assets is critical for estimating the amount of life insurance needed, and we prefer to tie our hands on this issue. In addition, general equilibrium allows us to calibrate the parameters of the model more effectively.

The specification of a fully-specified model allows us to clearly state what is meant by "adequate life insurance." Although this term is used repeatedly in the literature – especially in Bernheim *et.al* (2001,2002,2003), and Gokhale and Kotlikoff (2003) – it is not defined in terms of a calibrated general equilibrium model. Instead, those papers use a partial equilibrium decision problem with exogenous prices and a very specific utility function – Leontief over consumption across periods – to assess whether patterns are

puzzling.<sup>3</sup> We instead use more standard theory to assess the life insurance patterns. In particular, we wish to estimate how much life insurance is being held for consumption-smoothing purposes rather than the multitude of other motives. In the course of the estimation, we actually find that the amount of actuarially-fair life insurance held for consumption-smoothing purposes greatly exceeds the observed value from the data, despite the utility function being characterized by low amounts of curvature, and thus constitutes a puzzle.

Certain government policies may be substitutes for private life insurance. In particular, survivor benefits from social security are a potentially-important channel that reduces the amount of life insurance held in the data. In addition, conditional transfer programs like welfare, which pays only single mothers, could also act as substitutes for private life insurance policies. We introduce these policies into the model economy and find that these programs do not significantly reduce the amount of life insurance purchased, as holding patterns are very similar whether or not such programs exist; however, this result tends to strengthen our belief that life insurance purchases are puzzling.

Given our model, we make welfare calculations to determine the impact of a life insurance market. We find that welfare of newborns increases by only 0.02 percent if households have access to an actuarially-fair life insurance market. However, simulations that explore the importance of life insurance for particular groups – in particular, poor widows with large numbers of dependents – suggest larger benefits. Such groups do not hold a lot of life insurance, suggesting that the mismatch identified by Bernheim *et.al.* (2001,2003) may hold up under a more complete theoretical investigation.

The paper is organized as follows. We present the theoretical model and calibrate it to U.S. data. We present our results regarding three main issues. First, we estimate the extent to which consumption-smoothing motives drive life insurance decisions, both with and without government programs that act as substitutes. Second, we compute the welfare gains to newborns of having access to a life insurance market. And third, we explore the implications for consumption and leisure in households experiencing a death to the husband. We conclude with some comments about the model abstraction and future work.

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<sup>3</sup>Other papers that employ this term include Auerbach and Kotlikoff (1989,1990,1991).

## 2 The Model Economy

In this section, we describe our dynamic general equilibrium model. The decision making unit is the household, which may contain more than one individual. Households enter a period with a demographic state comprised of age, sex, size, and marital status; this state evolves stochastically over time. Within this environment, households make consumption-savings, labor-leisure, and portfolio decisions. In addition to the households, we have three other types of agents. Production firms rent capital and labor from households and produce a composite capital-consumption good. Insurance firms collect premium payments for life insurance policies and make payments to households. Finally, the government collects labor income taxes, consumes goods, and makes transfer payments to households in the form of welfare and social security.

### 2.1 The Demographic Structure

The economy is inhabited by individuals who live a maximum of  $I$  periods. The demographic structure of a household is a four-tuple that depends on age, the adult structure of the household, the marital status of the household, and the number of children in the household. Denote the age of an individual by  $i \in \mathcal{I} = \{1, 2, \dots, I\}$ . Survival probabilities are dependent on age and sex. The second element of the demographic variable is the adult structure of the household; we assume this variable can take on one of three values:  $p \in \mathcal{P} = \{1, 2, 3\}$ . If  $p = 1$ , then the household is made up of a single male. A value of  $p = 2$  denotes a household comprising of a single female, while  $p = 3$  denotes a household with a male and a female who are married.

The third element in the four-tuple is the marital status of the household. We define the marital status by  $m \in \mathcal{M} = \{1, 2, 3, 4\}$ . Four values are needed to account for various events that have an impact on the household. A value of  $m = 1$  denotes a household that is composed of a single adult, either male or female, that has never been married. If  $m = 2$ , then the household is comprised of a single individual that has become single due to a previous divorce. If  $m = 3$ , the household is a single individual that has been widowed. Finally,  $m = 4$  represents a married household.<sup>4</sup>

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<sup>4</sup>Some gender-marital status pairs are infeasible. The only pairs that are feasible are  $(p = 1, m = 1)$ ,  $(p = 1, m = 2)$ ,  $(p = 1, m = 3)$ ,  $(p = 2, m = 1)$ ,  $(p = 2, m = 2)$ ,  $(p = 2, m = 3)$ , and  $(p = 3, m = 4)$ .

The last element in the four-tuple denotes the number of children in the household. We denote this demographic state variable by  $x \in \mathcal{X} = \{0, 1, 2, 3, 4\}$ . This tells us that the household can have between zero and four children. We limit the number of children to four per household for computational reasons.<sup>5</sup> Single female households can bear children, but single male households cannot. We do not separately track the age of the children; rather, we assume that they age stochastically according to a process that leaves them in the household twenty years on average.

A household's demographic characteristics are then given by the four-tuple  $\{i, p, m, x\}$ . We will define a subset of demographic characteristics made up of the tuple  $\{p, m, x\}$  as  $\hat{z}$ ; this subset evolves stochastically over time. We assume that the process for these demographic states is exogenous with transition probabilities denoted by  $\pi_i(\hat{z}'|\hat{z})$ ; note that the transition matrix is age-dependent. To avoid excessive notation, we define the age specific transition matrices so that their rows add up to the probability of being alive in the next period. In constructing the transition matrix, a number of additional assumptions had to be made. In particular, marriage and divorce create some special problems. We assume that when a divorce occurs, the household splits into two households and economic assets are split into shares according to the sharing rule  $(\rho, 1 - \rho)$  where  $\rho$  is the fraction of household wealth allocated to the male. Any children are assigned to the female. If a household happens to die off (all parents die in a given period) we assume that the children disappear as well. For marriage, we only allow individuals of the same age to marry. In addition, a male with children and a female with children can only marry if the joint number of children is less than the upper bound. This set of assumptions and our demographic structure results in a relatively sparse transition matrix.<sup>6</sup>

The computation of this transition matrix is described in the appendix. The basic demographics of the calibrated population are presented in Table 1. We find that 68 percent of the population is currently married and 32 percent is single. Of the single households, divorced households make up 14 percent of the population, widowed households make up 7 percent of the population,

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<sup>5</sup>Actual data for number of children per female for 1999 indicates that the number of females with five or more children is less than 2.7 percent of females. By abstracting away from these households we are not ignoring a significant fraction of the population.

<sup>6</sup>The transition matrix for a specific age is  $(p, m, x) \times (p, m, x)$ . Out of this set of transition elements, only twenty-seven can be non-zero, plus the nonzero probability of transition into death.

and households which have never been married make up 10 percent of the population. When looking at children, we find 77 percent of households live with no kids, either because they have never had children or the children have left the household, 18 percent of households contain a single child, and about 5 percent have multiple children. This distribution exactly matches that found in the data.

## 2.2 The Household

### 2.2.1 Preferences

Household utility depends on the level of household consumption, male leisure, and female leisure. We specify the household utility function as

$$E_0 \sum_{t=1}^I \beta^{t-1} \frac{\left[ C_t^\mu (T_m - h_{mt})^{\chi(1-\mu)} (T_f - h_{ft} - \iota x_t)^{(1-\chi)(1-\mu)} \right]^{1-\sigma} - 1}{1 - \sigma}$$

where  $C_t$  denotes the level of household consumption,  $T_i$  is the time endowment,  $T_m - h_{mt}$  represents male leisure, and  $T_f - h_{ft} - \iota x_t$  defines female leisure. Female leisure differs from male leisure; female leisure depends on hours supplied  $h_f$  as well as a leisure cost per child captured by  $\iota x$ , where  $\iota \in [0, 0.25]$ .<sup>7</sup> In contrast, male leisure depends solely on hours supplied  $h_m$ . The remaining parameters in the utility function are the discount factor  $\beta \geq 0$ , the weight of household consumption in utility  $\mu \in (0, 1)$ , the relative weight of male leisure  $\chi \in (0, 1)$ , and the curvature parameter  $\sigma \geq 0$ .

Our utility function requires some discussion. The preference ordering that is represented by this utility function assumes that there is no disagreement over future states between married individuals, which would not generally be true in the presence of differential mortality rates, wages, and leisure costs. We finesse this problem by assuming that gender has no meaning within a marriage; that is, members of a married household do not know whether they are male or female. Further, each adult member views the probability of becoming the single male or the single female due to divorce as the same (50 percent), and therefore do not disagree about the value of

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<sup>7</sup>The upper bound is the maximum time endowment divided by the maximum number of children.

savings in those states. This assumption also ensures that preferences are not fundamentally altered by the death of an adult member.

The other complication related to our household utility function is the elasticity of substitution between male and female leisure. Mainly for computational purposes we have set this value to 1. There does not appear to be any accepted theory addressing the issue of the joint household decision for labor supply that applies to our model; hence, we lack any standard by which to judge our assumption outside the performance of the model itself. If we consider a specification with a constant elasticity of substitution that differs from one, the decision rules for labor supply are no longer linear and the model becomes computationally infeasible. Special cases that we can solve have grossly counterfactual implications for joint labor supply. For example, perfect complementarity implies that either no member or both members hit the lower bound of zero for labor supply, which does not correspond to observed patterns in the data. On the other extreme, perfect substitutability implies that females will only work in households with zero male leisure, clearly at odds with the data as well. By continuity, elasticities of substitution close to either extreme will generate similar counterfactual implications.

We define household consumption as

$$C_t = (1_{pt} + \eta x_t)^\theta c_t$$

where  $1_{pt}$  is an indicator function that takes on the value of 1 if the state variable  $p$  is either 1 or 2 or the value 2 if  $p$  is equal to 3, (*i.e.*, the married state),  $x_t$  is the state variable indicating the number of children in the family, and  $(\theta, \eta)$  are parameters. The parameter  $\theta \in [-1, 0)$  accounts for economies of scale in consumption, while the parameter  $\eta \in [0, 1]$  converts children into adult equivalents. Our utility function will enforce nonnegativity of consumption and leisure without imposing a constraint of that sort.

The labor-leisure decision in our environment will not be smooth – rather, it will feature a nonconvexity in the choice set for hours. To accommodate this feature, we assume that the time endowment is 1 for each member of the household, but supplying a positive amount of labor in a given period requires a fixed time cost of 0.02 units. In addition, we restrict the labor supply decision to involve the choice of supplying zero or more than 0.15 units of time to the market, with nothing in between.<sup>8</sup> We incorporate

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<sup>8</sup>0.15 units of labor supply corresponds to approximately halftime employment (20

this nonconvexity into the model economy because smooth versions did not produce the wealth equality between single and dual earner families observed in the data – dual earner families had close to twice as much wealth, which is counterfactual. That is, we have the choice for hours being

$$\begin{aligned} h_m &\in \mathcal{H}_m = \{0, [0.15, 0.98]\} \\ h_f &\in \mathcal{H}_f = \{0, [0.15, 0.98 - \iota x]\}. \end{aligned}$$

### 2.2.2 Household Environment

Households live in an uncertain environment that arises from demographic factors as well as a household specific productivity shock. Each period the household receives a productivity shock  $\epsilon \in \mathcal{E} = \{\epsilon_1, \epsilon_2, \dots, \epsilon_E\}$ .<sup>9</sup> In addition to the demographic state discussed above, the household begins a period with wealth  $a \in \mathcal{A}$ ; this space will be bounded from below by the requirement that consumption be nonnegative and bounded from above by the finiteness of the individual time horizon. The state for the household is the demographic situation, the productivity shock, and the wealth position:

$$s = (a, \epsilon, p, m, x, i).$$

Given this state, the household’s sources of funds are wealth and labor earnings. Labor earnings come from the hours worked by both males and females (if of working age) or government social security payments (if retired). Let  $h_i$  denote hours worked by the household member of gender  $i \in \{f, m\}$ . Each unit of labor pays  $w\epsilon v_i$  to the male worker and  $w\epsilon v_i \phi$  to the female;  $w$  is the aggregate wage rate,  $\epsilon$  is the age-independent idiosyncratic wage factor,  $v_i$  is the age-specific earnings parameter, and  $\phi \in (0, 1)$  corrects for the male-female wage gap. Let  $\varpi$  denote the social security payment,  $\varphi$  denote the welfare payment,  $\tau$  the payroll tax rate,  $1_\varpi$  an indicator of social security qualification, and  $1_\varphi$  an indicator of welfare qualification.<sup>10</sup> Total

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hours a week).

<sup>9</sup>We assume the productivity shock is household specific, meaning that both the husband and wife receive the same productivity shock. This assumption is made for computational purposes; given that the correlation between wages within a family is positive – see Guner and Knowles (2003) – this assumption is not unreasonable as a first approximation.

<sup>10</sup>That is, we have the indicator functions defined as

$$1_i = \begin{cases} 1 & \text{if the household qualifies for program } i \\ 0 & \text{otherwise} \end{cases},$$

labor income is then given by

$$H = (1 - 1_{\varpi})(1 - \tau) w \epsilon v_i (h_m + \phi h_f) + 1_{\varpi} \varpi + 1_{\varphi} \varphi.$$

With this level of funds, the household must consume and purchase assets. The only assets that are available are capital  $k$  and term life insurance policies  $l$ . The budget constraint for a household of age  $i$  is

$$c + k' + ql' \leq a + H \quad (1)$$

where  $q$  is the price of a life insurance policy.<sup>11</sup> We will discuss the details of the government transfer system in the calibration section.

The next period wealth level of a household  $a'$  depends on the capital and life insurance choices as well the future demographic state. If the household enters the period and remains in the same demographic state, the future wealth level is constrained by

$$a' \leq (1 + r') k' \quad (2)$$

where  $r'$  is the net return on savings.<sup>12</sup> If a divorce occurs in a household that starts the period in one of the married states, the male adult in the marriage has a wealth level next period equal to

$$a' \leq \rho(1 + r') k' \quad (3)$$

and the female adult has

$$a' \leq (1 - \rho)(1 + r') k', \quad (4)$$

where  $\rho \in (0, 1)$  is the sharing rule. If death of a spouse occurs, the wealth evolution equation is

$$a' \leq (1 + r') k' + l' \quad (5)$$

as the life insurance policy pays off. If a household enters as a single adult and becomes married, we have to merge the budget constraints of two single adult households. A marriage yields the wealth equation

$$a' \leq (1 + r') (k' + k'_{si}) \quad (6)$$

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where  $i = \{\varpi, \varphi\}$ .

<sup>11</sup>In our model, whole life insurance policies are equivalent to a portfolio of term life insurance policies and riskless capital, given that we abstract from tax issues.

<sup>12</sup>We employ the convention that a 'prime' on a variable denotes the value in the next period.

where  $k'_{si}$  is the age-dependent average capital for single households. The wealth of households that entirely die is estate-taxed at 100 percent and used to fund government expenditures.

Both life insurance and capital holdings are restricted to be nonnegative:

$$k', l' \geq 0.$$

We do not specifically model the reasons behind our asset market restrictions. For life insurance at least, appealing to adverse selection would probably suffice as a negative position in life insurance is equivalent to a long position in an annuitized asset. For capital, however, this restriction is somewhat more troublesome. We do not wish to complicate the model further by incorporating debt constraints.

The timing of events is important. We assume that divorce and marriage occur before death; that is, demographic changes occur first and then survival is determined. Furthermore, our demographic state only includes the last change; for example, households who get married, then divorced, then remarried, then widowed, are considered widowed. Fortunately, there will be only a small number of such households in equilibrium, and we do not feel the added burden involved in tracking past states to be worthwhile. Furthermore, we lack the individual data necessary to calibrate the transition matrix to these past events.

### 2.3 Aggregate Technology

The production technology of this economy is given by a constant returns to scale Cobb-Douglas function

$$Y = K^\alpha N^{1-\alpha}$$

where  $\alpha \in (0, 1)$  is capital's share of output and  $K$  and  $N$  are aggregate inputs of capital and labor, respectively. The aggregate capital stock depreciates at the rate  $\delta \in [0, 1]$  each period. Our assumption of constant returns to scale allows us to normalize the number of firms to one.

Given a competitive environment, the profit maximizing behavior of the representative firm yields the usual marginal conditions. That is,

$$r = \alpha K^{\alpha-1} N^\alpha - \delta \tag{7}$$

$$w = (1 - \alpha) K^\alpha N^{-\alpha}. \tag{8}$$

The aggregate inputs of capital and labor depend on the decisions of the various individuals in the economy. Let  $\Gamma$  denote the distribution of households over the states  $(a, \epsilon, p, m, x, i)$  in the current period. Aggregate labor input and capital input are defined as

$$N = \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} \epsilon v_i (h_m(a, \epsilon, p, m, x, i) + \phi h_f(a, \epsilon, p, m, x, i)) \Gamma(da, d\epsilon, p, m, x, i)$$

and

$$K = \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} a \Gamma(da, d\epsilon, p, m, x, i).$$

The goods market clears when

$$C + I + G = Y,$$

where  $G$  is aggregate government consumption.

## 2.4 The Life Insurance Firm

In this paper, we assume that the life insurance market is a perfectly competitive market. As a result, we can examine the behavior of the single firm that maximizes profits subject to a constant returns to scale technology. The price of insurance (the premium) will be determined by the zero profit condition in each period. We further assume that this firm accumulates no net worth, so that intertemporal pricing mechanisms are not operative.

We will consider an insurance firm that offers only term life insurance; we set the term to 1 period because the household always has the option to cease payment and terminate the contract.<sup>13</sup> The life insurance company sells policies at the price  $q$  and pays out to a household that loses a spouse. The price  $q$  can depend on the age and demographic characteristics of the household; however, we will not allow means-testing or history-dependence in these prices. Means-testing can be ruled out by allowing for unobservable storage technologies that enable a household to falsify observable wealth freely; history-dependence is ruled out entirely for computational reasons. To the extent that the transition matrix encodes some of the past outcomes that are relevant for current mortality, this restriction likely has little content.

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<sup>13</sup>We abstract from annual renewal pricing issues. Because life insurance markets are characterized by adverse selection problems which may be revealed over time, the price of renewals could differ from the price charged a first-time buyer.

Life insurance only pays off if an adult household member dies; we assume that the policy covers both members. Clearly, a critical aspect in the pricing of life insurance is the expected survival rate for an individual. We will represent the probability of an age  $i$  individual surviving to age  $i + 1$  as  $\psi_{p,m,x,i}$ . The zero profit condition for a life insurance firm is

$$\begin{aligned} & \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} (1 - \psi_{p,m,x,i}) \frac{1}{1+r'} l' \Gamma(da, d\epsilon, p, m, x, i) \quad (9) \\ &= \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} q(p, m, x, i) l' \Gamma(da, d\epsilon, p, m, x, i). \end{aligned}$$

The right hand side of this equation measures the revenue generated from the sale of life insurance policies to households. The left hand side measures the discounted payout due to deaths at the end of the period.

### 3 Stationary Equilibrium

We will use a wealth-recursive equilibrium concept for our economy and restrict ourselves to stationary steady state equilibria. Let the state of the economy be denoted by  $(a, \epsilon, p, m, x, i) \in \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I}$  where  $\mathcal{A} \subset \mathbb{R}_+$ ,  $\mathcal{E} \subset \mathbb{R}_+$ ,  $\mathcal{P} \subset \mathbb{R}_+$ ,  $\mathcal{X} \subset \mathbb{R}_+$  and  $\mathcal{M} \subset \mathbb{R}_+$ . For any household, define the constraint set of an age  $i$  household  $\Omega_i(a, \epsilon, p, m, x, i) \subset \mathbb{R}_+^5$  as all five-tuples  $(c, k', l', h_m, h_f)$  such that the budget constraint (2) and wealth constraints (3) – (6) are satisfied as well as the nonnegativity constraints.

Let  $v(a, \epsilon, p, m, x, i)$  be the value of the objective function of a household with the state vector  $(a, \epsilon, p, m, x, i)$ , defined recursively as

$$v(a, \epsilon, p, m, x, i) = \max_{(c, k', l', h_m, h_f) \in \Omega_i} \left\{ U \left( (1_p + \eta x)^\theta c, T_m - h_m, T_f - h_f - \iota x \right) + \beta E [v(a', \epsilon', p', m', x', i + 1) | a, \epsilon, p, m, x] \right\}$$

where  $E$  is the expectation operator conditional on the current state of the household. A solution to this problem is guaranteed because the objective function is continuous and the constraint correspondence is compact-valued and continuous.<sup>14</sup> However, since the constraint correspondence is

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<sup>14</sup>We are guaranteed compactness because the state space is bounded below by 0 and above by the future value of the highest finite realization of labor productivity times the maximum labor supply for the family.

not convex-valued, we cannot make definitive statements about the uniqueness of the solution or the properties of the value function.

**Definition 1** A *stationary competitive equilibrium* is a collection of value functions  $v : \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I} \rightarrow \mathbb{R}_+$ ; decision rules  $k' : \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I} \rightarrow \mathbb{R}_+$ ,  $l' : \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I} \rightarrow \mathbb{R}_+$ ,  $h_m : \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I} \rightarrow \mathcal{H}_m$ , and  $h_f : \mathcal{A} \times \mathcal{E} \times \mathcal{P} \times \mathcal{M} \times \mathcal{I} \rightarrow \mathcal{H}_f$ ; aggregate outcomes  $\{K, N\}$ ; prices  $\{q, r, w\}$ ; government policy variables  $\{\tau, \varpi, \varphi\}$ ; and an invariant distribution  $\Gamma(a, \epsilon, p, m, x, i)$  such that

- (i) given  $\{w, r, q\}$  and  $\{\tau, \varpi, \varphi\}$ , the value function  $v$  and decision rules  $c, k', l', h_m$ , and  $h_f$  solve the consumers problem;
- (ii) given prices  $\{w, r\}$ , the aggregates  $\{K, N\}$  solve the firm's profit maximization problem;
- (iii) the price vector  $q$  is consistent with the zero-profit condition of the life insurance firm;
- (iv) the goods market clears:

$$f(K, N) = \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} c \Gamma(da, d\epsilon, p, m, x, i) + K' - (1 - \delta) K;$$

- (v) the labor market clears:

$$N = \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} \epsilon v_i (h_m + \phi h_f) \Gamma(da, d\epsilon, p, m, x, i);$$

- (vi) the government budget constraint holds:

$$\begin{aligned} & \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} \varpi I_{\varpi} \Gamma(da, d\epsilon, p, m, x, i) + \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} \varphi I_{\varphi} \Gamma(da, d\epsilon, p, m, x, i) + G \\ &= \int_{\mathcal{A} \times \mathcal{E}} \sum_{\mathcal{P} \times \mathcal{M} \times \mathcal{X} \times \mathcal{I}} \tau (1 - I_{\varpi}) w \epsilon v_i (h_m + \phi h_f) \Gamma(da, d\epsilon, p, m, x, i); \end{aligned}$$

- (vii) letting  $T$  be an operator which maps the set of distributions into itself, aggregation requires

$$\Gamma'(a', \epsilon', p', m', x', i + 1) = T(\Gamma)$$

and  $T$  be consistent with individual decisions.

We will restrict ourselves to equilibria which satisfy  $T(\Gamma) = \Gamma$ .

## 4 Calibration

We calibrate our model to match features in the U.S. data. Our calibration will proceed as an exercise in exactly-identified Generalized Method of Moments where we attempt to match a small set of moments in the data. These moments will not involve the distribution of life insurance, however, so we will be able to ask the model questions about the size of the demand for life insurance for consumption-smoothing purposes, the main point of the paper.

We select the period in our model to be one year. First we examine the preference parameters in the model. The average wealth-to-GDP ratio in the postwar period of the U.S. is about three; this number will help pin down the value for the discount factor  $\beta$ . The average individual in the economy works about thirty percent of their time endowment; we use this number to help determine the parameter  $\mu$ . We also select  $\chi$  so as to match the ratio of the hours supplied by females to males. The 1999 Current Population Survey reports average annual hours worked for males in 1998 is 1,899 while average annual hours worked for females in the same period is 1,310; this ratio is used to help determine  $\chi$ , the relative weight of male leisure. These three targets are appended to the market clearing conditions and the entire system is solved numerically.

Other parameters we set directly. From time use surveys, we note that females allocate about 2 hours per day per child for care and females conduct about two-thirds of all such care, leading us to set  $\iota = 0.145$ . The relative wage parameter  $\phi$  is selected to be 0.77, consistent with estimates from the 1999 CPS on the relative earnings of males and females, and we set the divorce sharing rule to  $\rho = 0.5$ .<sup>15</sup> We use Greenwood, Guner, and Knowles (2001) to specify the first two of these parameters:  $\eta = 0.3$  and  $\theta = -0.5$ . Given little *a priori* consensus on the value of the curvature parameter, we choose  $\sigma = 1.5$ , a value which is consistent with choices typically made in the business cycle literature.<sup>16</sup> Choosing this value provides a lower bound for our estimate, since higher risk aversion (which is positively related to  $\sigma$ ) generates a higher demand for life insurance.

The technology parameters that need to be specified are determined by

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<sup>15</sup>We abstract from alimony and related post-divorce transfer payments for computational reasons.

<sup>16</sup> $\sigma$  turned out to have little impact on the distribution of life insurance holdings over the range [1.0, 4.0].

the functional form of the aggregate production function and the capital evolution equation. The aggregate production function is assumed to have a Cobb-Douglas form, since the share of income going to capital has been essentially constant. We specify labor’s share of income,  $1 - \alpha$ , to be consistent with the long-run share of national income in the US, implying a value of  $\alpha = 0.36$ . The depreciation rate is specified to match the investment/GDP ratio of 0.25, taken from the same data, yielding a value of  $\delta = 0.1$ .

The specification of the stochastic idiosyncratic labor productivity process is extremely important because of the implications that this choice has for the eventual distribution of wealth. Storesletten, Telmer and Yaron (2001) argue that the specification of labor income or productivity process for an individual household must allow for persistent and transitory components. Based on their empirical work, we specify  $\epsilon$  to evolve according to

$$\begin{aligned}\log(\epsilon') &= \omega' + \varepsilon' \\ \omega' &= \Psi\omega + v'\end{aligned}$$

where  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$  is the transitory component and  $\omega$  is the persistent component with  $v \sim N(0, \sigma_v^2)$ . STY estimate  $\Psi = 0.935$ ,  $\sigma_\varepsilon^2 = 0.01$ , and  $\sigma_v^2 = 0.061$ . Fernández-Villaverde and Krueger (2000) approximate the STY process with a three state Markov chain using the Tauchen (1986) methodology – this approximation yields the productivity values  $\{0.57, 0.93, 1.51\}$  and the transition matrix

$$\pi = \begin{bmatrix} 0.75 & 0.24 & 0.01 \\ 0.19 & 0.62 & 0.19 \\ 0.01 & 0.24 & 0.75 \end{bmatrix}.$$

The invariant distribution associated with this transition matrix implies that an individual will be in the low or the high productivity state just under 31 percent of the time and the middle productivity state 38 percent of the time. The age-specific component of income is estimated from earnings data in the PSID and produces a peak in earnings at real age 47. Note that with endogenous hours, this process (which is estimated from labor earnings) is not correct, but we checked that it produces an earnings distribution relatively consistent with the data despite the bias.

We assume that the mandatory retirement age is 65 (45 in model periods) and that agents live at most 100 years (80 model periods). We enforce retirement by setting the efficiency units of ages above 64 equal to zero.

Pricing in the life insurance industry is done relative to an individual who lives to be 100, so this horizon seems appropriate. Furthermore, a long retirement phase mitigates the impact of the terminal age on the behavior during the working ages. We require a relatively-short period to induce the persistent demographic states that give rise to significant demand for life insurance. The transition matrix for demographic states is difficult to construct. Due to the presence of history-dependence in the probabilities of marriage, divorce, mortality, and fertility, we found that we could not analytically construct this matrix. As a result, we used a Monte Carlo approach to generate the probability of transitioning between different states. In the computational appendix we detail the procedures followed to generate the transition matrix.

The last issue we must examine is the government income support system. We choose to set the labor income tax to  $\tau = 0.353$ , which is the average marginal tax rate on income plus payroll taxes and Medicaid. Social security benefits are set to be 0.4 percent of average earnings with survivor benefits that pay 60 percent of the deceased partner's wages; note that this means the female survivor will receive higher benefits than the male. Furthermore, survivor benefits are also paid to working age households with children at the same rate; these benefits drop to zero if children are not present in the household. To qualify for a welfare payment, the household must consist of a single mother who supplies no labor and has less than 1/3 average wealth; payments are equal to 17 percent of average earnings for each child in the household. Government consumption is computed as a residual value that sets the government budget constraint to equality.

## 5 Actuarially-Fair Insurance

We now detail our results. This section will consist of three subsections. In the first section, we explore the patterns of life insurance holdings in the model when all government policies are in place and pricing is **actuarially-fair**. We define this term to mean that the price that a household pays for a life insurance contract is exactly equal to the conditional probability that one (but not both) of the adult members of the household does not survive to the next period.

To focus the paper on a narrow set of questions, we restate some of the relevant facts. One, we wish to see what the model predicts for the total

amount of life insurance held; in the data (see Chambers, Schlagenhauf, and Young 2003) married households have 0.65 GDPs worth of life insurance. Given that the model has no incentives for singles to hold insurance, this number is the appropriate one to use as a benchmark. Two, the participation rate of married households is 68.7 percent with single and dual earners having participation rates of 67.7 and 69.5 percent respectively; in quantitative terms, they are essentially equal. Three, holdings by single and dual earners are approximately the same size: 3 times earnings. Four, the peak in participation rates lags the peak in holdings by 20 years. We wish to assess the extent to which these observations are driven by consumption-smoothing motives within the household.

## 5.1 Benchmark

Our calibration results are presented in Table 2. As can be seen, the interest rate  $r - \delta$  is around 1.16 percent per annum which is a reasonable value for risk-free government debt over the postwar US period, consistent with the average return to capital measured in McGrattan and Prescott (2003). Regarding the values for  $k_s$ , the model economy finds that the amount of capital held by singles is quite low; they hold about 13 percent of the total wealth in the economy. The wealth distribution produced by the model is broadly consistent with the data as well, especially along inequality dimensions; for example, the Gini coefficient in the model is 0.74, which is fairly close to that from the data. Furthermore, the ratio of the average to the peak of consumption, labor supply, and wealth also match those in the data. The model misses the upper tail of the income distribution, a common problem in overlapping generations economies with realistic wage processes, because no household can draw a sequence of labor productivity high enough to produce very large wealth positions.

In Figure 3, we plot the distribution of life insurance holdings by age from the model (we use five-year cohorts to eliminate high frequency noise from the data caused by sampling error). The peak in holdings occurs at age 30, well before the peak in the data (which is at age 50). The peak in the model coincides with the peak in the present value of future labor earnings (on average), and that consideration drives the very high life insurance/income ratios during the early periods of working life. The initial increasing segment is caused by a combination of rising present values for future labor income and the liquidity constraint. The model overstates

the demand for life insurance for purely consumption-smoothing concerns, because we are predicting LI/income ratios in excess of four at the peak, while the data peaks at only 2.5.

The participation rate for married households in the model is 59 percent, about 8 percent below that in the data. Our estimate suggests that a large number of households are using life insurance as a consumption-smoothing device (at least partially). Figure 4 presents the distribution of participation rates by age – it peaks at real age 33 at a value of 100 percent and declines over the rest of the lifecycle; by retirement age, only a very small fraction of households participate at all (but this number is not zero because there is some risk due to imperfect survivor benefits). In contrast, the data for married households peaks much later – age 70 – at 85 percent and remain above 60 percent across all ages.

In static insurance environments, it is straightforward to show that our utility function will imply that an agent will hold a positive amount of any fair insurance policy. However, our economy does not produce this result, as evidenced by the distribution of participation rates. Part of this hump-shape may be due to the government policies that act as life insurance, but as we will show in the next subsection not very much. More importantly, there is an interaction between the liquidity constraint and the insurance holdings. Young households who choose  $l' = 0$  in the model are liquidity constrained in capital; as a result, their marginal rate of substitution is too high relative to the interest rate. Furthermore, this inequality means that, in equilibrium, their marginal rate of substitution also does not get equated to the price of a life insurance contract, effectively forcing this agent to buy actuarially-unfair insurance. Since the fraction of agents who are liquidity-constrained initially declines with age, we observe rising participation rates over this range.

On the down side of the wage profile, age-specific productivity is beginning to fall and wealth is still rising, so that labor supply is falling, particularly for the less-productive females. But the constraint  $h_i \geq 0.15$  then will start to bind for some of these agents, pushing them off their first-order conditions along that dimension. Again, the binding of this constraint – which forces the marginal utility of consumption to be too high today – causes the household to face an effectively-unfair life insurance market, generating a drop in participation. If we eliminate the nonconvexity in the labor decision (as we did in previous versions of this paper), the participation rate does not decline over the later working years, instead displaying a discontinuous decline at retirement, suggesting that the corners are the key to generating

participation rates strictly below one.

The two results – holdings and participation – suggest that retirees have reasons for holding life insurance that are dominated by bequest motives or tax avoidance concerns, suggesting that social security is an effective public provider of life insurance. To discern whether this interpretation is correct, we eliminate the survivor benefits in the next subsection; we expect to see an increase in life insurance holdings by the retired households in response to a rise in the uncertainty about their transfer income.

## 5.2 The Effects of Survivor Benefits

We next explore the effects of altering the nature of the government policies that may be substitutes for life insurance. We eliminate the survivor benefit for social security – that is, the household receives  $\varpi$  if two adults are present but  $0.5\varpi$  if only one is present, regardless of the reason (never married, divorced, or widowed). Benefits to working age households who lose their spouse are also completely eliminated. Our intuition suggests that this change will increase the amount of life insurance purchased, particularly by the retired households, since they are now exposed to significantly more labor income risk.

The equilibrium changes very little when survivor benefits are eliminated – this result could be anticipated given that we are abstracting away from changes in tax policy that such changes would allow. Aggregate life insurance rises to 98 percent of GDP. The change in the distributions are in the expected direction – more holdings and larger participation rates – but the changes are quantitatively so small that aggregates are unaffected to four decimal places. Similar effects are present if welfare payments are eliminated, but these effects are even smaller. It does not appear that survivor benefits are critical to understanding the decisions relating to life insurance; for the remainder of the paper they will remain in place.

## 5.3 Welfare Gains

The results in the prior subsections suggest that the aggregate welfare gains emanating from the life insurance market might be large in the model, since agents in the model are purchasing large amounts of insurance. Our preferred approach for calculating welfare gains would be to use a transitional

dynamic approach, since we could make welfare statements about individuals. Unfortunately the computational burden of the model keeps us from using this approach. We therefore provide a crude estimate of the welfare gains by calculating the lifetime expected welfare gains associated with a newborn person getting to live in the economy with life insurance versus being forced to live in an economy without that market.<sup>17</sup>

We define the *ex ante* welfare of a newborn individual as:

$$W = \int_{\epsilon} \sum_p v(0, \epsilon, p, 1, 0, 1) \pi_{\epsilon}^{inv} \pi_p. \quad (10)$$

Consistent with newborns, the age is 1, the initial asset position is zero, and the number of children is zero. If the newborn is male,  $p = 1$ , while a newborn female would be characterized by  $p = 2$ .  $\pi_{\epsilon}^{inv}$  denotes the invariant distribution of  $\epsilon$  while  $\pi_p$  is the probability of being born gender  $p$ . We compute welfare under a version of the model without operative life insurance markets; denote this welfare value by  $W_0$ . We then compute the lifetime percent increase in consumption  $\lambda$  needed to make an individual indifferent between that world and the one with operating life insurance markets. Given the utility function, this increase solves the equation

$$W_1 = (1 + \lambda)^{\mu(1-\sigma)} W_0 \quad (11)$$

where  $W_1$  is average newborn utility in an economy with life insurance markets.  $\lambda$  thus measures the welfare gain associated with life insurance assets.<sup>18</sup>

The equilibrium outcomes from the economy without operative life insurance markets is essentially identical to the benchmark model. Compared to this economy, we find that having access to a life insurance market that is priced actuarially-fairly yields a welfare gain of 0.2 percent of consumption. Oddly, the gain from having access to a life insurance market in the economy without survivor benefits is smaller: 0.1 percent of consumption. These calculations are on the same order of magnitude as those in Lucas (2003) for the welfare costs of consumption fluctuations, numbers which are universally considered small. The reason for such small numbers here is

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<sup>17</sup>This calculation is not too misleading since the aggregate capital stock is essentially the same across the two model economies.

<sup>18</sup>Note that, since we have incomplete markets, we cannot be sure that introducing additional assets will increase welfare. Such perverse outcomes are associated with very strong general equilibrium effects, which our previous results show are not present in our economy.

that most agents end up fairly well-insured over the lifecycle. The ones who are typically poorly-insured – the young – face very little mortality risk and are also disproportionately single. Other poorly insured agents, such as single mothers with many children, are rare events from the perspective of a newborn and thus bad outcomes in those states get very little weight.

The aggregate number above can be quite misleading, however, when heterogeneity is present. As mentioned above, we would prefer to compute individual-specific welfare costs based on wealth, productivity, and demographics. Such computations are impossible given the size of the model environment. However, we suspect that the welfare gains are concentrated in certain groups, in particular the poor and middle-aged widows who have large numbers of children. In an attempt to identify these groups, we use our model to conduct a series of simulations that examine how a household is impacted by a death of a spouse over their remaining life cycle. We consider a household who is impacted by a death of a wage earner when a life insurance market is present or not, paying particular attention to the impact of a death on the average paths for consumption and hours worked. Specifically, we take a household in a particular state of the world and simulate the effect of losing the male adult member of the household, averaging over 5000 sample impulse responses to create the expected effect.

For these experiments, we will concern ourselves only with poor households. Wealthy households self-insure effectively without having access to a life insurance market, and thus the absence of that market is of limited relevance to them. We explore the impact of being widowed when the family has limited resources during middle age, both with a small number of children and a large number (1 versus 4). Our finding here is that both groups appear to benefit from the presence of a life insurance market and that the benefit is increasing in the number of children present. Calculations based on households being in other states of the world are qualitatively similar, but the gains appear to be largest for this group.

### **5.3.1 Poor Households with One Child**

We first consider a household with a low wealth level - less than half average wealth – that is 40 years old. Such a household is really in much worse shape than it may first appear, since at age 40 they are in the middle of their prime earning years and have very little wealth relative to their cohort. As a result, the household cannot self-insure against the unexpected loss of a wage earner

effectively. A death in this household will likely have large ramifications for consumption-saving and labor-leisure decisions and the availability of a life insurance market may be quite important.

Figure 5 shows the paths for consumption and labor supply for this household. Overall, since the effective discount rate is negative, consumption will be rising over the lifetime of the household, and this is clearly evident in the consumption paths. In the period after the shock, consumption for the household in the LI economy rises while that in the no LI economy falls. Consumption remains higher in the LI economy until retirement (the large spike in consumption at retirement is due to the massive increase in leisure evident in the second row of panels). Although not shown on the graph, the standard deviation of consumption also falls when life insurance is available, so that this measure understates the true gain. As for labor supply, the household in the no LI economy must increase hours by much more than the one in the LI economy, and this increase is permanent (until retirement of course), but in each case labor supply is declining from its initial value.

The reason consumption falls in the no LI case but not in the LI case involves the total wealth of the household, where total wealth is measured as the sum of current financial wealth and the present discounted value of all future labor income plus transfers. If there is no life insurance market, wealth unambiguously falls since the maximum labor income during working ages is cut by more than half and transfers are reduced on net (survivor benefits generally do not replace enough to cover the losses). As a result, consumption and leisure both decline. When given access to life insurance, the household can mitigate the loss of wealth caused by reduced time endowment by generating an increase in current financial wealth through life insurance payments; this results in the smaller increase in labor supply. Relative to a female whose husband did not die, the widow in the LI economy still has less financial wealth and less consumption – these households are not fully-insuring themselves against mortality risk.

### 5.3.2 Poor Households with Four Children

The situation is more extreme for a poor household that has 4 children, the maximum allowed for in the model. The behavior of consumption is not qualitatively different; however, the magnitude of the lost consumption is larger. It is in the labor supply decision that the main difference arises. In both economies, a poor household with 4 children will not be supplying

positive female hours. When the death shock occurs, the no LI widow must increase labor supply much more over the course of her remaining life – the peak at retirement age is 3 times as high. Given that childcare costs are nearly 80 percent of the time endowment at the beginning of this simulation, increasing labor supply is very costly for this widow. Again, we find that the wealth of the widow lies below that of an equivalent nonwidow.

## 6 Conclusion

Our model has examined the life insurance portfolio decisions of households in a model with a reasonable amount of demographic detail. Our estimate is that the consumption-smoothing motive for holding life insurance is potentially very large, so large in fact that it constitutes a puzzle from the perspective of economic theory. We find that, in an actuarially-fair environment, married households would hold life insurance equal to twice that in the data despite having no bequest or tax avoidance motives. We see this as a major puzzle, and we provide supporting evidence to that in Bernheim *et.al.* (2001, 2002, 2003) that the buying patterns in this market seem difficult to rationalize.

There are omitted motives for holding life insurance that may be relevant to our discussion, but we argue that they would increase not decrease the puzzle. First, the data contain a large number of \$5000 policies, held disproportionately by the elderly. These policies are "death policies" that cover the average cost of a funeral in the US.<sup>19</sup> Such policies are clearly for consumption-smoothing, and as such they would increase the amount of policies held in the model and also drive the participation rate up to nearly 1, deepening the puzzle. Second, many divorce settlements require life insurance equal to the present value of future alimony payments – again, this would increase holdings in the model. Furthermore, it would provide single males a motive for holding life insurance that is in fact related to consumption-smoothing. Since the model's prediction for married holdings exceeds the total held in the data, adding this feature would only make this market more at odds with existing theory.

Theoretically, there are modifications to the model environment that would potentially change our estimates. For example, habit formation

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<sup>19</sup>Source: BEA.

in consumption would dramatically increase the demand for consumption-smoothing via life insurance, as the household views the drops in consumption evident in our model with strong distaste. Thus, we expect our results to be robust to those types of preference modifications, as well as changes due to increased risk aversion. Myopic behavior or excessive short-run discounting – as in Laibson (1994) – might have the opposite effect, reducing demand sufficiently that it no longer constitutes a puzzle, but such models are difficult to work with and we prefer to stay closer to established theory.<sup>20</sup>

Allowing for endogenous marriage and divorce might also change our results, but this margin is unlikely to be important enough to overturn the answers here. Households who perceive a high probability of divorce at the end of the period will be less likely to buy life insurance, but this effect will be of second-order importance if calibration is done to match average flows.<sup>21</sup> For this reason, we suspect our answers will also be robust along this dimension. Finally, we have abstracted from a very important source of disposable income variation, particularly for retirees – unexpected medical expenses. As seen in Livshits, MacGee, and Tertilt (2003), such shocks can be quite large and would be expected to increase in variance as the household ages. Adding this source of uncertainty into the model would increase the demand for life insurance since assets would be needed for precautionary purposes much more so than they are now.

Our model also has shown that demographic shocks combined with wage uncertainty is capable of producing a wealth concentration that close to that observed in US data. That is, consistent with evidence in Cubeddu and Ríos-Rull (2002), we argue that the composition of families matter, as divorces, children, and premature deaths are major shocks to the wealth and income of a household. The study of public programs which could provide insurance against these shocks, like alimony, no-fault divorce, and child support, is clearly possible within our framework.

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<sup>20</sup>Excessive optimism of the sort considered in Brunnermeier and Parker (2003) would likely operate in the same way, producing a "it won't happen to me" aversion to life insurance. A similar effect would be expected under the "denial of death" approach of Kopczuk and Slemrod (2005).

<sup>21</sup>The issue of the interrelation between wealth and the endogenous marriage/divorce decision is studied in Guner and Knowles (2003), among other places.

## 7 Computational Appendix

This appendix details the computational strategy used to solve the model. The appendix is divided into four parts. First, we discuss the computation of the household problem; we use backward induction along the lifetime to solve for the value function. Second, we discuss the generation of the invariant distribution over wealth, productivity, demographics, and age. Third, we discuss our method for computing market clearing prices and the solution to calibration equations. Fourth, we detail our Monte Carlo method for computing the transition matrix for the demographic states.<sup>22</sup>

The basic algorithm is as follows:

1. Guess values for the vector of wealth for single individuals  $k_{si}$  and the rental rate  $r$ .
2. Solve the consumer's problem and obtain the value function  $v$  and the decision rules  $k'$ ,  $l'$ ,  $h_m$ , and  $h_f$ . This step involves building a nonlinear approximation to the value function and is described in detail below.
3. Iterate on an initial distribution of idiosyncratic states until convergence. This step assumes that the distribution of  $a$  is over only a finite number of points and redistributes mass iteratively. To conserve on computational time, we calculate the invariant distribution over stochastic states and use this information to start the iterations on the distribution of wealth.
4. Check that the values for  $r$  and  $k_{si}$  agree with those in step 1. If not, then update and return to step 1.

When calibrating the model, we add to step 1 guesses for the discount factor  $\beta$ , the consumption weight  $\mu$ , and the relative male leisure weight  $\chi$ . We then check whether our guesses imply the right values for the wealth/GDP ratio, the average hours worked, and the ratio of female to male labor supply. We do not need to check the profit condition of the life insurance company,

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<sup>22</sup>Fortran 95 code to solve for this equilibrium is available at <http://garnet.acns.fsu.edu/~eyoung/programs>. This code does not implement the parallel solution method and thus is appropriate for casual users, but runtimes are extremely long. The program's search for the equilibrium price and parameter vector also requires a significant amount of babysitting.

since it will earn zero profit at every point in the price space even with operating costs, given that we assume everyone pays the same surcharge. Also, since we are assuming that government consumption is determined residually, we do not need to check the government budget constraint.

## 7.1 Solving the Household Problem

We will now discuss the solution of the household's problem. Let current wealth  $a$  lie in a finite grid  $A \subset \mathbb{A}$ . We must solve a two-dimensional continuous portfolio problem in  $(k', l')$ ; furthermore, to complicate the problem both face short-sale constraints and the price of life insurance is small, leading to flat objective functions in the portfolio space. As a result, we take the approach used in Krusell and Smith (1997) and Guvenen (2001) to solve the problem. To begin, we guess that the agent holds zero life insurance. We then find the optimum level of savings in capital by solving the Kuhn-Tucker condition

$$\begin{aligned} & (1_p + \eta x)^{\theta\mu(1-\sigma)} c^{\mu(1-\sigma)} (1 - h_m)^{\chi(1-\mu)(1-\sigma)} (1 - h_f - \iota x)^{(1-\chi)(1-\mu)(1-\sigma)} \times \\ & \left( \frac{\mu}{c} \left( -1 + \frac{\partial h_m}{\partial k'} wv_i \epsilon + \frac{\partial h_f}{\partial k'} \phi wv_i \epsilon \right) - \frac{\chi(1-\mu)}{1-h_m} \frac{\partial h_m}{\partial k'} - \frac{(1-\chi)(1-\mu)}{1-h_f-\iota x} \frac{\partial h_f}{\partial k'} \right) \\ & + \beta E[v_1(a', \epsilon', m', i+1)](r+1-\delta) \\ & \leq 0 \end{aligned}$$

where  $h_m$  and  $h_f$  solve

$$\begin{aligned} \frac{\mu w v_j \epsilon}{a + w v_i \epsilon (h_m + \phi h_f) - k' - q l'} &= \frac{\chi(1-\mu)}{T_m - h_m} \\ \frac{\mu w v_i \epsilon \phi}{a + w v_i \epsilon (h_m + \phi h_f) - k' - q l'} &= \frac{(1-\chi)(1-\mu)}{T_f - h_f - \iota x}; \end{aligned}$$

this equation is solved via bisection. If  $h_i$  fails to satisfy the lower bound 0.15, we set it to that value. Next, we let life insurance holdings be slightly positive:  $l' = 0.0001$ . If this increase reduces lifetime utility, the agent has zero life insurance optimally. If not, we use bisection to locate the correct value for  $l'$ , increasing  $l'$  whenever the gradient at the optimal value for  $k'$  is positive and decreasing it whenever the gradient is negative. We repeat this process for zero labor supply for the female and for both members – it can be shown that the male member of a married household will never set labor

supply to zero if the female supplies a positive amount so we have only three cases to check. The value function is then set equal to the max over these three cases.

Ignoring bequests, we assume that

$$v(\cdot, \cdot, \cdot, \cdot, I + 1) = 0.$$

Then, for each  $i \leq I$  and using  $v(\cdot, \cdot, \cdot, \cdot, i + 1)$  as the value function for the next age, we can obtain the value function for this age as the solution to

$$v(a, \epsilon, p, m, i) = u(C^*, h_m^*, h_f^*) + \beta E[v(a^*, \epsilon', p', m', i + 1)].$$

Cubic spline interpolation is used whenever we need to evaluate  $v(\cdot)$  at points not on the grid for  $a$ .

## 7.2 Computing the Invariant Distribution

For the invariant distribution, the procedure outlined in Young (2002) is employed. For each idiosyncratic state and age vector  $(a, \epsilon, p, m, i)$  we compute next period's wealth contingent on demographic changes. After locating  $a'(a, \epsilon, p, m, i)$  in the grid using the efficient search routine **hunt.f** from Press *et.al.* (1993), we can construct the weights

$$A(a, \epsilon, p, m, i) = 1 - \frac{a'(a, \epsilon, p, m, i) - a_k}{a_{k+1} - a_k}$$

where

$$a' \in [a_k, a_{k+1}].$$

Now consider a point in the current distribution

$$\Gamma^n(a, \epsilon, p, m, i).$$

This mass is moved to new points according to the following process. For each set  $(\epsilon, p, m, i) \times (\epsilon', p', m')$  we calculate the probability of transition; denote this value by  $\rho(\epsilon, p, m, i, \epsilon', p', m')$ . Mass is distributed then to the point

$$\Gamma^{n+1}(a_k, \epsilon', p', m', i + 1)$$

in the fraction

$$A(a, \epsilon, p, m, i) \rho(\epsilon, p, m, i, \epsilon', p', m') \Gamma^n(a, \epsilon, p, m, i)$$

and to the point

$$\Gamma^{n+1}(a_{k+1}, \epsilon', p', m', i + 1)$$

in the fraction

$$(1 - A(a, \epsilon, p, m, i)) \rho(\epsilon, p, m, i, \epsilon', p', m') \Gamma^n(a, \epsilon, p, m, i).$$

Looping this process over each idiosyncratic state and age computes the new distribution. This process continues until the change in the distribution is negligible. Note that we can compute the weights and the brackets before iteration begins; since these values do not change we can store them and use them as needed without recomputing them at each step.

### 7.3 Solving for Market Clearing and Calibration

We now discuss how we solve for the equilibrium, given the algorithms for computing the value function and the invariant distribution. This algorithm takes the following form:

1. Take the fitness functions to be the sum of the squared deviations of the equilibrium conditions. We then attempt to solve

$$\min_{\omega} \{\langle F(\omega), F(\omega) \rangle\}$$

where  $\omega$  is a vector of prices and parameters,  $F$  is the vector-valued function of equilibrium conditions, and  $\langle \cdot \rangle$  is the inner product function. For the initial calibration this vector is of dimension 4:

$$[r, \beta, \chi, \mu].$$

It turns out that the wealth held by singles can be computed *ex post*, then the model resolved once at that vector, without affecting the other variables in the system.

2. Set an initial population  $\Omega$  which consists of  $n$  vectors  $\omega$ . Given our strong priors on the values for certain variables, we do not choose this population at random. Rather, we concentrate our initial population in the region we expect solutions to lie.

3. Evaluate the fitness of each member of the initial population.
4. From the population, select  $n$  pairs with replacement. These vector-pairs will be candidates for **breeding**. The selection criterion weights each member by its fitness according to the rule

$$1 - \frac{\langle F(\omega_j), F(\omega_j) \rangle}{\sum_{j=1}^n \langle F(\omega_j), F(\omega_j) \rangle}$$

so that more fit specimens are more likely to breed.

5. From each breeding pair we generate 1 offspring according to the BLX- $\alpha$  crossover routine. This routine generates a child in the following fashion. Denote the parent pair by  $(\omega_i^1, \omega_i^2)_{i=1}^4$ . The child is then given by

$$(h_i)_{i=1}^4$$

where  $h_i \sim \text{UNI}(c_{\min} - \alpha I, c_{\max} + \alpha I)$ ,  $c_{\min} = \min\{\omega_i^1, \omega_i^2\}$ ,  $c_{\max} = \max\{\omega_i^1, \omega_i^2\}$ , and  $I = c_{\max} - c_{\min}$ . Our choice for  $\alpha$  is 0.5, which was found to be the most efficient value by Herrera, Lozano, and Verdegay (1998) in their horse-race of genetic algorithms for a sum-of-squares objective function like ours.

6. We then introduce **mutation** in the children. With probability  $\mu_G = 0.15 + \frac{0.33}{t}$ , where  $t$  is the current generation number, we mutate a particular element of the child vector. This mutation involves 2 random numbers,  $r_1$  and  $r_2$ , which are  $\text{UNI}(0, 1)$  and 1 random number  $s$  which is  $N(0, 1)$ . The element, if mutated, becomes

$$h_i = \begin{cases} h_i + s \left[ 1 - r_2^{\left(1 - \frac{t}{T}\right)^\delta} \right] & \text{if } r_1 > 0.5 \\ h_i - s \left[ 1 - r_2^{\left(1 - \frac{t}{T}\right)^\delta} \right] & \text{if } r_1 < 0.5 \end{cases}$$

where we set  $\delta = 2$  following Duffy and McNelis (2001). Note that both the rate of mutation and the size shrinks as time progresses, allowing us to zero in on potential roots.

7. Evaluate the fitness of the children.
8. From each family trio, retain the most fit member. We now are left with exactly  $n$  members of the population again.

9. Compare the most fit member of the last generation, if not selected for breeding, with the least fit member of the new generation. Keep the better of the two vectors. If the most fit member of generation  $t - 1$  is selected for breeding this step is not executed. This step is called **elitism** and is discussed in Arifovic (1994).
10. Return to step 4 unless the population's average fit has not changed significantly across generations.
11. After obtaining a good calibration vector, subsequent equilibria are computed using Brent's method to find a zero in the equilibrium condition for the capital market.

Note that some parameter values are not permitted; for example,  $\mu$  cannot be larger than one or less than zero. In these cases the fitness of a candidate is assumed to be  $10^6$ ; that is, a large penalty function is attached to impermissible combinations. These candidates will be discarded immediately and never breed.

In our implementation of the genetic algorithm, we parallelize computation by attempting to send each separate evaluation of  $F(\omega)$  to a separate processor. For the genetic algorithm, each generation requires  $n$  evaluations for the new offspring (the parents have already been computed). This can be very costly when conducted serially, so we exploit parallel coding and multithreading to the extent that it is possible.

## 7.4 Monte Carlo Generation of Transition Matrix

The transition matrix for the demographic states turned out to be impossible to write down analytically. The problem is that we wish to remain faithful to the Census data on mortality, marriage, divorce, and fertility. To do so requires that the transition probabilities be dependent on the path taken to a particular state; for example, it matters for mortality of women how many children they have had, not just the number that they currently have, due to the inherent health risks associated with childbirth. Also, large numbers of children typically are associated with lower income families who have higher mortality rates as well. We were not able to construct the matrix analytically as a result, since any given current demographic state could have a very large number of histories associated with it. Therefore, we chose the following Monte Carlo approach.

To begin, we draw a random UNI (0, 1) random variable; if below 0.495 the new household is a male, if not it is a female. We then check whether the household dies, gets married, bears children, or survives unchanged, using data from the US Census and CDC to determine age and gender specific transition probabilities. We truncate the number of children to 4 (which leaves out less than 2.7 percent of the population), we do not allow for multiple births within 1 year, and single males cannot have children (no adoption). In cases of divorce, the children proceed with their mother, and if the last adult in the household dies, all the children living in the household die as well. Given the data and these assumptions, we then let the household age 1 year and repeat the process until death. This procedure is repeated 60 million times; the transition matrix is then estimated using the sample probabilities. Due to sampling error (even with this gigantic number of observations), some states are rarely encountered in the simulation, which leads to some irregularities in the transition matrix used in the program.<sup>23</sup>

This sampling error introduced by our Monte Carlo approach to calculating the transition matrix is not innocuous. Small irregularities in the mortality rates generate large irregularities in life insurance holdings since the premium paid by an individual is tied down by their mortality rate. Thus, we are careful to generate death probabilities which match the observed data. That is, the small dip in the death probability of males around age 30 is actually observed in the data. To insure the correct probability of death, we normalize the transition matrix to the correct death probability. Each row of the matrix is divided by the simulated survival probability and then multiplied by the true survival probability. Each row contains the true survival probability and a smooth death probability is observed over the life cycle.

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<sup>23</sup>Matlab code to generate this matrix is available at <http://garnet.acns.fsu.edu/~eyoung/programs>.

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**Table 1**  
Demographics of Simulated Economy

Characteristic	Percent of Population
Married	68.02
Single	31.98
Divorced	7.25
Widowed	14.49
Never Married	10.24
0 Kids	76.63
1 Kid	18.83
2 Kids	4.30
3 Kids	0.20
4 Kids	0.01

**Table 2**  
Calibration Results

Variable	Value
$r$	0.116
$\beta$	1.005
$\mu$	0.310
$\chi$	0.575

Figure 1

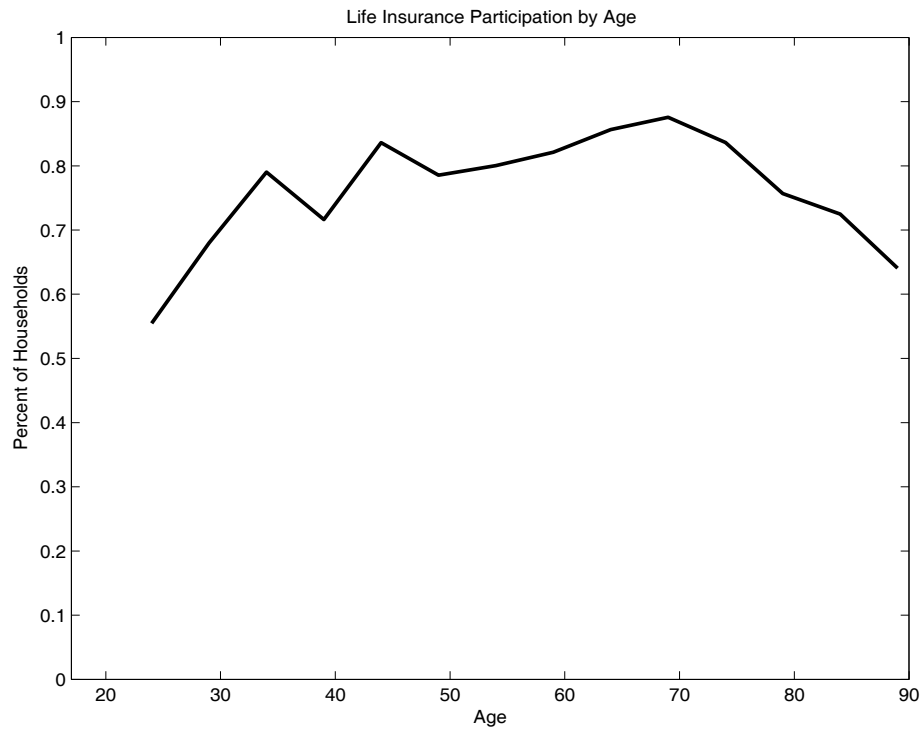


Figure 2

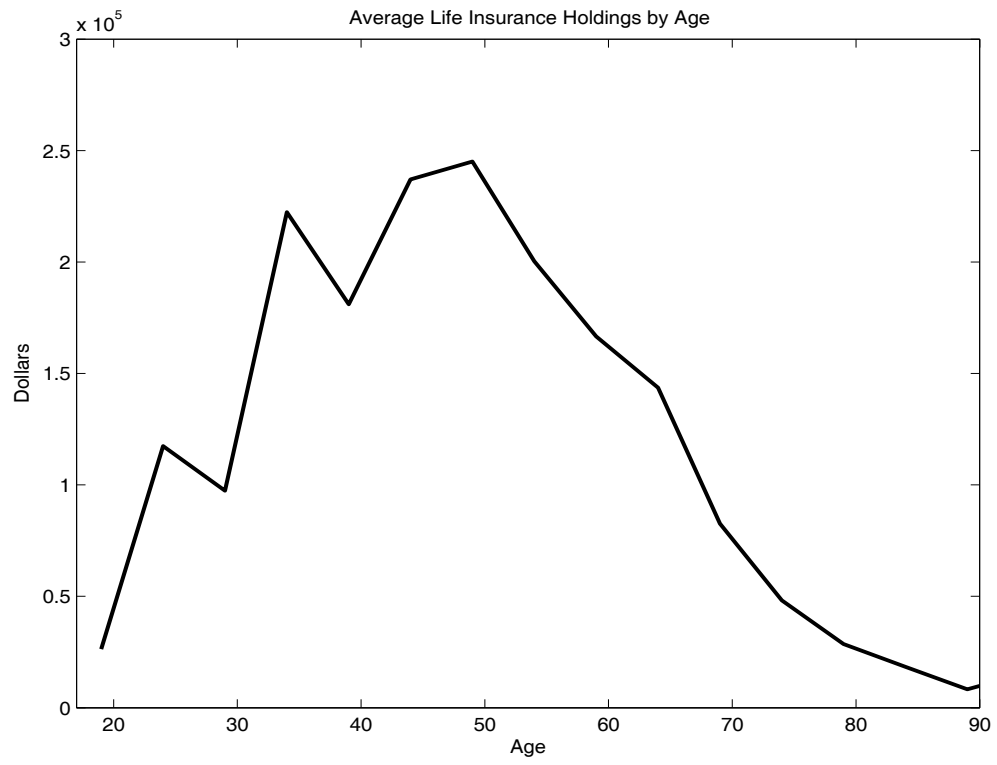


Figure 3

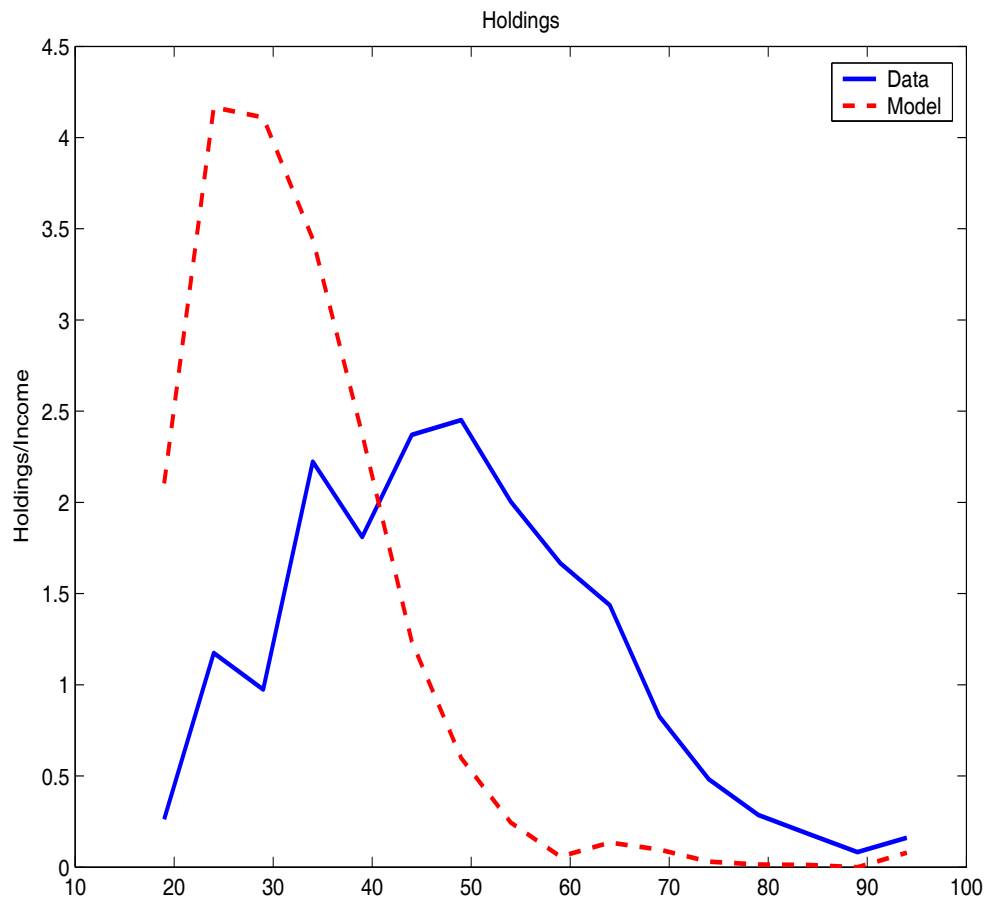


Figure 4

Participation Rates

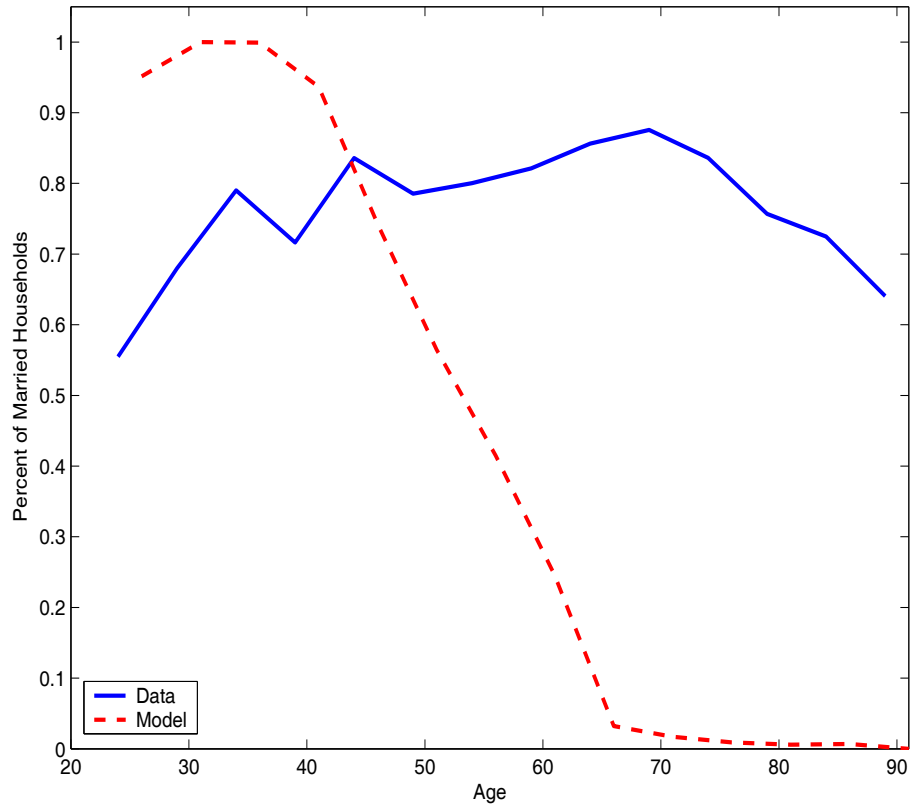


Figure 5

