MI 3750

Problem Set # 1 Solutions

1 a. $P(C_1) = 1/2$

b. $P(C_{5,6}) = 1/6 + 1/6 = 1/3$

c. $P(C_5) = 1/4$

d. $P(c) = P(0 < x < 1/3) = 1/3$

e. $P(A) = \frac{9/8}{4} = 9/32$

$P(B) = 1 - 9/32 = 23/32$

.2 $P(c) = \frac{\pi (1/2)^2}{\pi (1)} = \frac{1}{4}$

1.3 $P(HT) = 1/4$

$P(HT \cup TH) = 1/2$

1.4. a. $A_1 \cup A_2 = \{0, 1, 2, 3, 4\}$

$A_1 \cap A_2 = \{2\}$

b. $A_1 \cup A_2 = \{x : 0 < x < 3\}$

$A_1 \cap A_2 = \{x : 1 \leq x < 2\}$

c. $A_1 \cup A_2 = \{(x, y) ; 0 < x < 3, 0 < y < 3\}$

$A_1 \cap A_2 = \{(x, y) ; 1 < x < 2, 1 < y < 2\}$

1.5 a. $A^* = \{x : 0 < x < 5/8\}$

b. $A^* = \{(x, y, z) : x^2 + y^2 + z^2 < 1\}$

c. $A^* = \{(x, y) : x^2 + y^2 < 2$

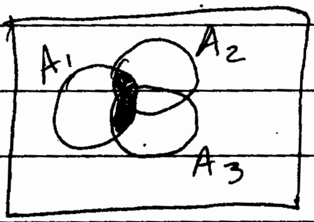
$\cap \{(x, y) : |x| + |y| > 2\}$

1.6. $A_1 \cup A_2 = \{$mary, army, rmay, ramy, amry, myra, myar, mray, ~~mrya~~, mayr$\}$

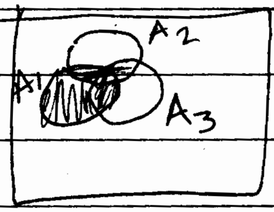
$A_1 \cap A_2 = \{$mary, mray$\}$

PS #1

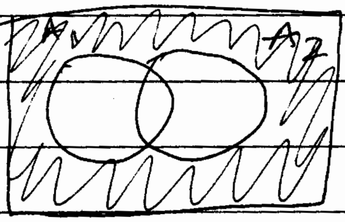
1.7 a.



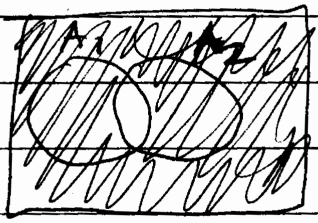
b.



c.



d.



1.22 $P(C_1) = 1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3$

$P(C_2) = 4/6$

$P(C_1 \cap C_2) = 2/6 = 1/3$

$P(C_1 \cup C_2) = 6/6 = 1$

1.23 $P(C_1) = 13/52 = 1/4$

$P(C_2) = 4/52 = 1/13$

$P(C_1 \cap C_2) = 1/52$

$P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$

$= 13/52 + 4/52 - 1/52$

$= 16/52 = 4/13$

1.24 $P(C) = \sum \left(\frac{1}{2^n}\right)$

$P(C_1) = 1/2 + 1/4 + 1/16 + 1/32 = 31/32$

$P(C_2) = 1/2^5 + \frac{1}{2^6} = 3/64$

$P(C_1 \cap C_2) = 1/32$

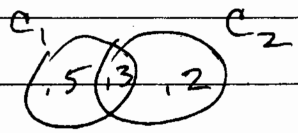
$P(C_1 \cup C_2) = 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64$

$= 63/64$

PS #1

1.25 $P(C_1) = 0.8$ $\qquad P(C_1 \cap C_2) = 0.3$

$P(C_2) = 0.5$



1.26 $P(C) = \int_4^\infty e^{-x} = -e^{-x}\Big]_4^\infty$

$= \frac{-1}{e^\infty} + \frac{1}{e^4} = \frac{1}{e^4}$

$P(C^*) = 1 - \frac{1}{e^4}$

$P(C \cup C^*) = \frac{1}{e^4} + \left(1 - \frac{1}{e^4}\right) = 1$

1.27 $\int_C e^{-|x|} \qquad -\infty < c < \infty$

$\int_{-\infty}^0 e^x + \int_0^\infty e^{-|x|} = 2$

Constant $= 1/2$

1.64 a. $\frac{\binom{6}{4}}{\binom{16}{4}} = .0082$ $\qquad$ b. $1 - .0082 = .9918$

c. $\frac{\binom{6}{1}\binom{3}{2}\binom{7}{1} + \binom{6}{2}\binom{3}{1}\binom{7}{1} + \binom{6}{1}\binom{3}{1}\binom{7}{2}}{\binom{16}{4}}$

$= 0.45$

1.65 $P(W') = \frac{990}{1000} \cdot \frac{989}{999} \cdot \frac{988}{998} \cdot \frac{987}{997} \cdot \frac{986}{996} = 0.9509$

$1 - P(W') = 1 - 0.9509 = 0.0491$

1.66 a. $\frac{\binom{13}{6}\binom{13}{4}\binom{13}{2}\binom{13}{1}}{\binom{52}{13}} = 0.00196$

b. $4\left(\frac{\binom{13}{13}}{\binom{52}{13}}\right) = 6.3 \times 10^{-12}$

PS #1

1.68 a. $\left(\frac{2}{8}\right)\left(\frac{1}{7}\right)(3) = \frac{6}{56}$

b. $\binom{5}{2}$

1.71 $\dfrac{\binom{13}{x}\binom{39}{13-x}}{\binom{52}{13}}$ $\qquad$ $\dfrac{\binom{13}{x}\binom{13}{y}\binom{26}{13-x-y}}{\binom{52}{13}}$

$x=2, y=5 \Rightarrow \dfrac{\binom{13}{2}\binom{13}{5}\binom{26}{6}}{\binom{52}{13}} = 0.0364$

1.73 a. $1 - \dfrac{\binom{2}{0}\binom{48}{5}}{\binom{50}{5}} = 0.1918$

b. $1 - \dfrac{\binom{2}{0}\binom{48}{x}}{\binom{50}{x}} \Rightarrow \dfrac{x}{50+49} = \dfrac{1}{2} \Rightarrow x = \sqrt{1225}$

$x = 35$

$50 - 35 = 15$

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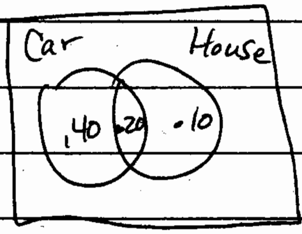
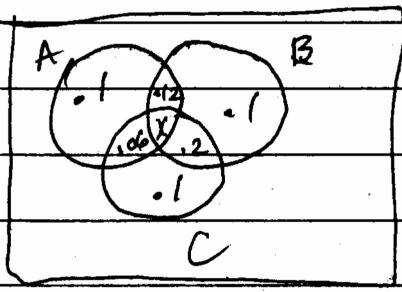
Problem Set #2 Solutions

1. 

$.6 + .3 - .2 = .70$

2. 

3. a. $\frac{102/937}{312/937} = \frac{102}{312} = 0.327$

b. $\frac{108/937}{625/937} = \frac{108}{625} = 0.173$

4. a. $.08 \times .15$
$+ .16 \times .08$
$+ .45 \times .04$
$+ .31 \times .05 = 0.0583$

b. $\frac{.08 \times .15}{.0583} = 0.2058$

5. $E(x) = 0 \times .7 + 200 \times .2 + 1000 \times .06$
$+ 5000 \times .03 + 10{,}000 \times .01$
$= 350$

$E(x^2) = 0^2 \times .7 + 200^2 \times .2 + 1000^2 \times .06$
$+ 5000^2 \times .03 + 10000^2 \times .01$
$= 1{,}818{,}000$

$\sigma = \sqrt{1{,}818{,}000 + 350^2} = 1302$

skew $= \frac{119\,3345000}{\sigma^3} = 5.40552013089$

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Problem Set 3 Solutions

1.a. $E(x) = .1(0) + .4(500) + .3(25000)$
$+ .15(100000) + .05(250000)$
$= \$35,200$

$Var(x) = E(x^2) - (E(x))^2 = 3,573,560,000$

$\sigma = 59,779.2606$

b. $E(x^3) = 0^3(.1) + 500^3 \times .4 + 25000^3 \times .30$
$+ .15 \times 100000^3 + 250000^3 \times .05$
$= 9.339 \times 10^{14}$

skewness $= 4.3812 = \dfrac{E(x^3)}{\sigma^3}$

c. Not binomial because more than 2 possible outcomes.
Not Poisson because mean and variance not equal.

2.a. $P(x=5) = \binom{40}{5}(.25)^5(.75)^{35}$
$= .02723$

b. $P(x \le 10) = p(x=10) = \binom{40}{10}(.25)^{10}(.75)^{30} = .1443$
$+ p(x=9) = \binom{40}{9}(.25)^9(.75)^{31} = .13971$
$+ P(x=8) = \binom{40}{8}(.25)^8(.75)^{32} = .11788$
$+ P(x=7) = \quad \vdots \quad = .0857$
$+ P(x=6) = \quad \vdots \quad = .05295$
$+ P(x=5) = \quad \vdots \quad = .02723$
$+ P(x=4) = \quad \vdots \quad = .01135$
$+ P(x=3) = \quad \vdots \quad = .00368$
$+ P(x=2) = \quad \vdots \quad = .00087$
$+ P(x=1) = \quad \vdots \quad = .00013$
$+ P(x=0) = \binom{40}{0}(.25)^0(.75)^{40} = .0001$

$P(x \le 10) = .5839$

PS #3

2.c. $P(X \geq 20)$

$= 1 - P(X \leq 19)$

$= 1 [P(X=19) + P(X=18) + P(X=17) +$
$P(X=16) + P(X=15) + P(X=14) +$
$P(X=13) + P(X=12) + \ldots P(X=0)]$

$=$ ~~[illegible]~~

$= 1 - .9998$

$= .000175$

2.d. $E(X) = np = 40(.25) = 10$

$Var(X) = 40(.25)(.75) = 7.5$

$\sigma = 2.7386$

3.a. $E(X) = \left(0 \times {}^{6}/_{48}\right) + \left(1 \times {}^{12}/_{48}\right) + \left(2 \times {}^{16}/_{48}\right)$

$+ \left(3 \times {}^{8}/_{48}\right) + \left(4 \times {}^{3}/_{48}\right) + \left(5 \times {}^{2}/_{48}\right)$

$+ \left(6 \times {}^{1}/_{48}\right) + 7\left({}^{0}/_{48}\right)$

$= 2 = \lambda$

3.b. $\dfrac{e^{-2}2^{1}}{1!} = .2707$

3.c. $\dfrac{e^{-2}2^{0}}{0!} + .2707 + \dfrac{e^{-2}2^{2}}{2!} = .6766$

3.d. $1 - .6766 - \dfrac{e^{-2}2^{3}}{3!} = .1430$

3.e. $.1804 + \frac{e^{-2} 2^4}{4!} + \frac{e^{-2} 2^5}{5!} + \frac{e^{-2} 2^6}{6!} = .3135$

3.f. $1 - (.3135 + .6766) = .0099$

could be rounding differences here

4-7. $.0041(10{,}000)$
$+ .0730(1{,}000) = 114$

4-8. ~~E(X)~~ $114(10) + 50 = 1{,}190$

4-~~B~~. $\sqrt{.0041(10{,}000-114)^2 + .073(1{,}000-114)^2 + .9229(0-114)^2}$

$= 3{,}427.8418$

4-16. $E(X) = .276$
$E(X^2) = \frac{1}{15000}(3100 + 1540 + 2430)$
$= \frac{5450}{15000} = .363$
$\sigma = \sqrt{.363 - .276^2} = .536$

5-3. $P(X=3) = (.023)^3(.977)^9\binom{12}{3}$
$= .00217$

5-9. $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= .994$

5-16. a. $P(X=1) = \frac{e^{-.6}(.6)^1}{1} = .329$

b. $P(X>1 = 1 - [P(X \leq 1)] = 1 - [P(X=1) + P(X=0)] = .12$

PS #3

5-17. a. $P(X=0) = \dfrac{e^{-1.5}(1.5)^0}{0!} = .2231$

b. $P(X=1) = \dfrac{e^{-1.5}(1.5)^1}{1!} = .3347$

c. $P(X=2) = \dfrac{e^{-1.5}(1.5)^2}{2!} = .251$

5-18 $E(x) = .38\ (5000) = 1{,}900$

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Problem Set #4 Solutions

1.

Value	P
0	.69
1	.23
2	.07
3	.01

a. $0(.69) + 1(.23) + 2(.07) + 3(.01)$
$= 0.40$

b. $500(E(x)) + 50$
$500(4) + 50 = 250$

c. $E(x^2) = 0^2(.69) + 1^2(.23) + 2^2(.07) + 3^2(.01)$
$= 0.60$

d. $E(x^3) = 0^3(.69) + 1^3(.23) + 2^3(.07) + 3^3(.01)$
$= 1.06$

2. a. $E(\sqrt{W}) = \sqrt{50000}\,(.69) + \sqrt{40000}\,(.23) + \sqrt{30000}\,(.07) + \sqrt{20000}\,(.01) = 213.8273$

b. $E[\ln(W+10)] = .69\ln(50{,}010) + .23\ln(40{,}010) + .07\ln(30{,}010) + .01\ln(20{,}010) = 10.72376$

c. $E[\log(W-10)] = .69\log(49990) + .23\log(39990) + .07\log(29990) + .01\log(19990) = 4.6482$

3. a. $E[\ln(W)] = .7\ln 500000 + .2\ln 499800 + .06\ln 499000 + .03\ln 495000 + .01\ln 490000 = 13.1217$

PS #4

3. b. $\ln(W_I - X) = 13.12$

$$500000 - X = e^{13.1217}$$

$$x = 500000 - e^{13.1217}$$

$$x = \$351.58$$

6-6. $E(x) = \mu'_x(t=0) = \lambda e^t e^{\lambda(e^{t}-1)}$

$$= \lambda$$

$$E(x^2) = \mu''_x(t=0) = e^{\lambda(e^{t}-1)}(\lambda e^t + (\lambda e^t)^2)$$

$$= \lambda + \lambda^2$$

$$Var(X) = E(X^2) - (E(x))^2$$

$$= \lambda + \lambda^2 - \lambda^2 = \lambda = E(x)$$

6-15. $n = 20$ $\binom{20}{x}(.40)^1(.60)^{20-x}$

$p = .40$ $E(x) = 8$

6-16 a. $5(.4) = 2$ c. $x < .5 = 2$

b. $5(.4) = 2$ d. 2

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Problem Set 5 Solutions

12. $z = \frac{x-\mu}{\sigma} = \frac{27-20}{5} = 1.40$

15. $F(x) = 1 - e^{-\lambda x} = 1 - e^{-1/.7(1)} = .76035$

$\lambda = \frac{1}{.7}$

16. $P(X = .80) = F(x) = 1 - e^{-\lambda x} \Rightarrow \lambda = \frac{1}{5} = .2$

$.80 = 1 - e^{-.20x} = 1 - \frac{1}{e^{.20x}}$

$-.20 = \frac{-1}{e^{.20x}}$

$.20 = \frac{1}{e^{.20x}}$

$.20e^{.20x} = 1$

$e^{.20x} = 5$

$(1.2214)^x = 5$

$\frac{x \ln 1.2214}{\ln 1.2214} = \frac{\ln 5}{\ln 1.2214}$

$x = 8.04728$

17. $P(X \le 10) = F(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-[\ln(x)-\mu]^2/2\sigma^2}$

$= \frac{1}{10\sqrt{2\pi(3)^2}} e^{-[\ln(10)-5]^2/2(3)^2}$

$= \frac{1}{75.21398046} \cdot \frac{1}{1.49814076}$

$= \frac{1}{112.6811296} = 0.0088746$

[redacted]